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Supplier selection for development of petroleum industry facilities, applying multi-criteria decision making techniques including fuzzy and intuitionistic fuzzy TOPSIS with flexible entropy weighting

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ABSTRACT

Generic supplier selection from the perspective of multi-criteria decision making (MCDM) methodologies including crisp, fuzzy and intuitionistic fuzzy analysis of decision matrices has received much attention, but less so specifically for the gas and oil industry, and in terms of comparing performance of a number of available techniques. A set of 30 criteria are identified for assessing supplier selection for facilities and field development projects across the petroleum industry. Bidders are assessed in terms of these criteria, with varying degrees of uncertainty and subjectivity, using linguistic scoring terms that are then transformed into crisp and fuzzy numerical sets. Eight MCDM scoring methods are described mathematically and applied to a facilities-procurement scenario in order to analyze a linguistic-assessment matrix for five alternative bidders using the 30 recommended criteria. These scoring methods are: linear; non-linear; *the order of preference by similarity to an ideal solution (*TOPSIS); Fuzzy TOPSIS (with and without entropy weighting); and, intuitionistic fuzzy TOPSIS (IFT) with three alternative methods for calculating entropy weighting (W_e). Performance of the eight methods is assessed by comparing calculated rankings for the five bidders in relation to the defined supplier selection scenario for a base case and ten sensitivity cases. The results of the analysis suggest that entropy weightings applied to fuzzy sets provide more consistent bidder selection, and led to the proposal of a new *intuitionistic-fuzzy-TOPSIS-method-with-flexible-entropy-weighting* method that enables the entropy weighting scale to be tuned to suit the circumstances of specific scenarios using equation 30 to flexibly normalize the entropy weighting scale.

1. Introduction

Selecting suppliers, contractors and service providers through competitive bidding processes is a critical activity for most operating gas and oil operating organizations. For large facilities projects, such as the components of engineering, procurement and construction (EPC) contracts the value of the decisions involved on individual contracts can reach billions of dollars, and frequently involves value of hundreds of millions of dollars. Consequently, making appropriate decisions, and justifying and documenting the reasons for selecting a specific bidder rather than one of the competing bidders, is a process high on the agenda of investors and decision makers.

In fact supplier selection for large EPC-type and related contracts is a multi-faceted and multi-dimensional process. Although the bidders may be primarily bidding a price for supplies/services to be provided in conjunction with a specified contract of work package, typically there are multiple criteria, in addition to the bid price, that are involved in accessing the suitability of specific bidders to perform the work and associated with performing the tasks required. These criteria are presented here and categorized to establish the multi-criteria characteristics typically involved in supplier selection in the gas and oil industries.

Significantly the criteria that require assessment associated with such decisions range from those for which quantitative information is available to make objective comparisons among bidders, to qualitative information associated with significant uncertainties that typically involves subjective and/or speculative assessment. Hence, decision-making techniques are required that can incorporate quantitative, semi-quantitative and qualitative information, with varying degrees of associated uncertainty, and

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integrate them into a systematic, transparent and repeatable process.

Multiple criteria and multiple bidders are just two dimensions that need to be considered in the supplier selection decision-making process. In addition, there are often multiple decision makers, or analysts involved in the decision process. This is typically the case for large organizations (e.g. multi-disciplined departments and divisions influencing the decisions) and joint venture operations (i.e., assets where an operating company and one to several non-operating partner companies with equity interests all hold voting rights in major decisions associated with the asset held). In fact, it adds two key dimensions to the decision-assessment process: 1) different decision makers/analysts may apply weights to the multiple criteria associated with the decision differently; and, 2) based on voting rights or organizational structure the project operator may assign different levels of importance to the criteria ranking of certain decision-makers/analysts in formulating a combined-assessment on which the final decision is to be made.

Multi-criteria decision making (MCDM) techniques suitable to assess high-value supplier selection decisions need to take into account all of the above dimensions and issues if they are to be useful tools for gas and oil industry decision makers. MCDM analysis techniques include a range of well-developed and applied methodologies, such as *simple additive weighting* (SAW) (e.g., Churchman et al., 1957; Chen, 2012; Memariani et al., 2009), *analytic hierarchy process* (AHP) (e.g., Saaty, 1980; Junior et al., 2014), *elimination and choice expressing reality* (ELECTRE) (e.g., Roy, 1968; Saracoglu, 2015), *preference ranking organization method for enrichment evaluations* (PROMETHEE) (e.g., Behzadian et al., 2010; Vetschera and Teixeirade Almeida, 2012) and *the order of preference by similarity to an ideal solution* (TOPSIS) (e.g., Hwang and Yoon, 1981; Hwang and Lin, 1987; Taylan et al., 2014; Yurdakul and Ic, 2005), which is the focus of much of this study.

All of the methods mentioned can be applied to multi-discipline, group-decision-making situations applied to complex issues involving high levels of uncertainty. These MCDM methods make it possible to take into account different perspectives and decision preferences of several distinct decision makers. Distance-based-uncertainty techniques (e.g., TOPSIS) offer flexible tools for MCDA, because they easily permit decision makers to alter the criteria weights applied in the analysis, are relatively easy to compute and facilitate the identification of the most critical weights. Several studies have compared the performance of the different MCDM methods applied to specific situations (recent examples are: Thor et al., 2013 and Junior et al., 2014). Thor et al. (2013) concluded that TOPSIS outperformed SAW, AHP and ELECTRE methods when applied to plant maintenance design options because of its structured consistency, ease of calculation and its ability to handle large data sets. Junior et al. (2014) compared the application of fuzzy AHP and fuzzy TOPSIS methods to supplier selection and concluded that the Fuzzy TOPSIS method is superior in terms of ease of operation and its ability to handle greater numbers of criteria and suppliers and applying changes to criteria assessments. It is for the above reasons that this study focuses upon TOPSIS methods.

In this study various versions of the popular TOPSIS methodologies are used to evaluate the multi-dimensional supplier selection scenario from an MCDM perspective, and compare their performance with simple linear and non-linear summed scoring systems. The fundamental concept of TOPSIS is to select the alternative that has the shortest geometric distance from the "positive ideal" solution and the longest geometric distance from the "negative ideal" solution. It is, therefore, a distance-based uncertainty technique, which can be made flexible in the way in which weights are calculated (i.e. objective weights), combined with selective weights in some cases, and applied to the multiple criteria involved (e.g., Hyde et al., 2005; Hyde and Maier, 2006). Shih et al. (2007) demonstrated that TOPSIS could be readily extended to address group decision making, which is relevant to supplier selection in the gas and oil sector. For instance, in large organizations considering investment decisions associated with large projects, decisions are rarely taken by isolated departments or individual decision makers within that organization. It is more typical in practice for such decisions to be made as a collaborative effort by a group of decision makers, often with each decision maker contributing to the group decision applying distinctive weights to the criteria under consideration.

There are three clear types of MCDM methodologies commonly applied: 1) those using only crisp numbers; 2) those applying fuzzy numbers (Hsu and Chen, 1996); and, 3) those applying intuitionistic fuzzy sets (IFS) (Atanassov, 1999). Crisp numerical and verbal scoring systems typically do not reflect accurately real-world situations and the subjectivity of human judgments, which involve elements of vagueness, preferential biases, prejudice and general uncertainty, i.e., circumstances best described mathematically in terms of "fuzziness". TOPSIS can also be readily combined with fuzzy logic (i.e., fuzzy set theory, e.g., Zadeh, 1965, 1971; fuzzy arithmetic, e.g., Keufmann and Gupta, 1991; Zimmermann, 1991) in various ways to better reflect variable levels of uncertainty associated with the criteria assessments (e.g., Deng, 1999; Majd et al., 2014; Shapiro and Koissi, 2013). In addition to a conventional fuzzy set approach, TOPSIS can also incorporate intuitionistic fuzzy set (IFS) concepts, which are designed to provide more focus on degrees of indeterminacy and vagueness.

Fuzzy and IFS TOPSIS methodologies typically vary in the way in which criteria weights are calculated and applied. Weights can be subjective (i.e., based on decision-maker preferences and subjective judgments) or objective (i.e., mathematically derived from the numerical assessment information contained in the decision matrices), or a combination of both. IFS TOPSIS methods facilitate the combination of subjective and objective weights (Chen and Li, 2010) with the objective weights typically derived through the calculation of "entropy", i.e., not the term referred to in thermodynamics, but a so-called measure of fuzziness defining the degree of impression and vagueness contained within matrix data (Shannon, 1948; De Luca and Termini, 1972). Fuzziness describes the availability of less than perfect information caused by the inability to distinguish clearly whether elements belong or do not belong to a particular set. The calculated entropy measure should reveal or clarify how far a fuzzy set is from a crisp set (i.e., a set of non-fuzzy numbers) (Collan et al., 2015). De Luca and Termini (1972) developed the probabilistic entropy concept proposed by Shannon (1948) for non-probabilistic applications in fuzzy sets. Szmidt and Kacprzyk (2001) proposed the IF entropy measure, which is widely applied in IFS methods for calculating entropy-based weights, and other measures developing their principles are also applied (e.g., Vlachos and Sergiadis, 2007; Parkash et al., 2008; Ye, 2010). The established IFS approach is to use the calculated entropy value to develop entropy-based weights with which to adjust an IFS decision matrix. The general rule is that the better an attribute/criterion can discriminate a set of data (i.e., the lower its entropy value should be) the higher the entropy weight that set of data should be allocated. Hence, there is an inverse correlation between the calculated entropy value of a data set and the entropy-based weight applied to that data. The lower the entropy weight that is applied to an attribute/criteria, the lower the contribution that attribute/criterion makes to the decision selection (Wang et al., 2007; Wang and Lee, 2009).

Various methodologies have been applied to supplier selection scenarios across a range of industries in recent years. For general reviews of the various MCDM methods applied to supplier selection scenarios see DeBoer et al. (2001) and Wu and Barnes (2011). Jadidi et al. (2008) and Shahanaghi & Yazdian (2009) proposed integrated fuzzy TOPSIS methods to address supplier selection scenarios. Kahraman et al. (2009) applied fuzzy TOPSIS to group decision making in the selection of information system providers. Amiri (2010) proposed a combined AHP and Fuzzy TOPSIS method for oil field development project selection. Toloei Eshlaghy and Kalantary (2011) proposed a modified TOPSIS method for supplier selection. Tabar and Charkhgard (2012) suggested a Fuzzy TOPSIS model, integrated with the analytical network process to provide weights, for supplier selection and supply chain management decisions. Junior et al. (2014) compared fuzzy TOPSIS and Fuzzy AHP methods applied to supplier selection in the automotive production chain, concluding that fuzzy TOPSIS performed better in scenarios involving changing numbers of criteria and suppliers. Whereas, Yazdani (2014) proposed an integrated AHP – Fuzzy TOPSIS approach for green supplier selection in the automobile supply chain. Rouyendegh and Saputro (2014) applied a fuzzy TOPSIS model to address supplier selection for a fertilizer and chemical company. These studies highlight that supplier selection is an important issue spanning many industries and that addressing the issue with MCDM methods including TOPSIS is a topical area of research in which significant progress has been made in recent years. Many of the findings established from such detailed case studies described for a specific industry have more generic applications worthy of consideration by other industries, including gas and oil.

Most of the studies cited in the previous paragraph propose a methodology supported by a worked example, but do not conduct detailed performance comparisons with other established MCDM techniques, nor do they conduct sensitivity analysis on the impact of applying different subjective and objective weightings to the scenarios described. This study aims to address these issues as they relate to supplier selection scenarios in the gas and oil industry.

This study applies linear, non-linear, TOPSIS, fuzzy TOPSIS and IFS TOPSIS methodologies, with various entropy weighting calculations included in the latter three methods, to a supplier selection scenario considering criteria relevant to an oil and gas facilities development project. Comparisons, using sensitivity analysis associated with importance weightings assigned to three decision makers, are made among the eight MCDM methodologies described in terms of their suggested rankings among five bidders. The first three methods involve crisp numbers; the last five methods involve fuzzy logic designed to capture uncertainty in the analysis. In addition to the objective entropy weights two other subjective weights are applied to the scenario: criteria weights (W_c) applied by three decision makers; importance weights (W_g) applied to the assessments of each of the decision makers in a integrated analysis. Eight methodologies structured to incorporate W_c and W_g weighting flexibilities (with four also incorporating W_e weighting, and five involving fuzzy logic to factor in uncertainty) are developed here and applied to the supplier selection scenario involving five bidders and thirty criteria, assessed initially using simple linguistic terms. It is a relatively straightforward process to translate linguistic assessments into numerical, semi-quantitative scales, but the selection of those scales is itself subjective and does influence the ranking outcomes of the methodologies applied. The findings lead to recommendations about how MCDM methodologies might best be applied in relation to supplier selection scenarios in the gas and oil industry and the application of a new *intuitionistic-fuzzy-TOPSIS-method-with-flexible-entropy-weighting* methodology. The novelty of the proposed methodology lies in the introduction of a tuning capability to the derivation of the entropy weight scale via the use of an S factor, and in the use of that S factor to provide additional sensitivity analysis to decision ranking.

2. Supplier performance criteria relevant to the gas and oil industries

There are many criteria that require consideration in a generic supplier selection process, but there are a number of quite specific criteria that are relevant to many international gas and oil and gas procurements. Large EPC contracts for example often have the attention of governments, regulators, communities and non-governmental organizations, and perhaps, more so than many other industries, need to include local content to satisfy local procurement rules. Tables 1a–1c list 30 criteria considered relevant to the specifics requirements of the industry. It would certainly be possible to break these down this list further and/or add additional points for consideration. However, for the purposes of comparing methodologies, thirty criteria are considered adequate to identify the multi-criteria nature of the supplier selection process.

The thirty identified criteria are referred to by numbers only in the following analysis. Some general observations to make about the criteria identified are:

1. Each supplier selection exercise may have unique geographic and organizational factors that will result in modifications/additions to the thirty criteria identified, A MCDM methodology needs to be flexible enough to adjust the number of criteria to suit project requirements.
2. Several of the criteria listed are hard, if not impossible, to quantify accurately, e.g. local content, community relations, willingness to share risk, etc. This dictates that a qualitative assessment is required, at least as a starting point, for an assessment to accommodate all of the criteria listed.
3. There is likely to be a significant amount of uncertainty associated with assessments of many of the criteria. Assessment and scoring systems applying crisp numbers are unlikely to capture that uncertainty in the analysis. The subjective and inadequate/conflicting information available for some criteria and mixture of qualitative and quantitative information can make it hard to find and justify a "best choice" simply in terms of crisp numbers. In this context the term "crisp" is used to refer to a single point number, i.e., integer or real, that is not a fuzzy number, i.e., part of a fuzzy set (e.g., Aliev et al., 2015; Buckley, 2005).
4. The decision process typically has to consider many strategic and operational factors in addition to the criteria information, e.g., organizational strategic objectives, constraints and availability of certain resources, which can further complicate the decision-makers' task.
5. For most of the criteria higher outcomes mean better performance, whereas for other criteria lower outcome means better performance (i.e., criteria 1, 13, 24, 25 and 26 in Table 1). Certain scoring systems and methodologies applied need to take account of such differences in criteria characteristics.

3. Bidders, decision makers, preferences and importance

For the comparison of MCDM methodologies a supplier selection scenario involving five competing bidders is considered here (i.e., EPC1 to EPC5). An assessment of these five bidders, based on the thirty criteria identified (Table 1) is conducted by three decision

Table 1a
Multiple criteria to consider for gas and oil industry supplier selection grouped into finance, reputation and local content categories.

Multiple criteria to consider for supplier selection in gas and oil facilities projects		
Sector	Criteria numbers	Criteria details
Finance	1	Contract bid price/material costs
Finance	2	Supplier's/contractor's financial strength
Finance	3	Access to finance & financial guarantees (ECAs)
Finance	4	Payment schedule & credit terms
Finance	5	Retention payment & warranty terms
Reputation	6	Track record/reputation/professionalism
Reputation	7	Previous global experience
Reputation	8	Previous local experience & procurement compliance
Local focus	9	Local content (workforce/suppliers)
Local focus	10	Community/NGO relations and CSR
Local focus	11	Government (local & national) relations

Table 1b
Multiple criteria to consider for gas and oil industry supplier selection grouped into project management, supply chain and technical/engineering categories.

Multiple criteria to consider for supplier selection in gas and oil facilities projects		
Sector	Criteria numbers	Criteria details
Project management	12	Management and organizational framework
Project management	13	Work force turnover and experience
Project management	14	IT Software communication sophistication
Project management	15	Relationships/procurement with subcontractors
Supply chain	16	Logistics and supply base capabilities
Supply chain	17	Supply chain management
Supply chain	18	Equipment & resources availability
Technical/Engineering	19	Engineering and design expertise
Technical/Engineering	20	Access to appropriate technologies and patents
Technical/Engineering	21	Innovation/problem solving/flexibility

Table 1c
Multiple criteria to consider for gas and oil industry supplier selection grouped into risk mitigation and quality/performance categories. HSSE refers to health, safety, security and environment.

Multiple criteria to consider for supplier selection in gas and oil facilities projects		
Sector	Criteria numbers	Criteria details
Risk mitigation	22	Risk management & contingency planning
Risk mitigation	23	Willingness to share risk
Risk mitigation	24	Contract risk for sponsor
Risk mitigation	25	Time overrun risk
Risk mitigation	26	Cost overrun risk
HSSE	27	Safety, compliance & security record
HSSE	28	Environmental record
Quality/Performance	29	Quality control and standards
Quality/Performance	30	Operational plant reliability/handover expectation

makers (i.e., DM1, DM2 and DM3). It is assumed that those three decision makers weight the criteria differently (Table 2). Whereas, DM1 weights all criteria equally (i.e. weight applied to each criteria = 1/30 = 0.3333), DM2 weights criteria 1, 2, 6, 8 and 30 five times higher than the other criteria (Table 2). On the other hand DM3 weights criteria 1, 3, 7, 9, 11 and 29 at a higher level, criteria 10, 11, and 29 at an intermediate level, and the remaining criteria at a low level. Hence, even if these three decision makers agree on the assessment of each bidder in relation to a specific criterion, they wish to apply different weightings collectively to those assessments.

The supplier selection scenario also considers the importance assigned to the assessment of bidder by applying additional importance weighting factors. The base case scenario assumes equal weightings (i.e., 1/3 = 0.3333) applied to each decision maker's assessment. However, sensitivity cases are evaluated in which a range of unequal weightings, all summing to one, are applied. From a calculation perspective this means that three separate evaluations, one for each decision maker, are necessary, with the final decision integrating those three evaluations into a final assessment in such a way that the importance weightings are applied.

4. Applying a linguistic assessment and converting it into appropriate numerical scores

A simple five category linear scoring system is applied to the supplier selection scenario considered. This is given in Table 3 along with numerical scores that are to be applied by the MCDM methodologies considered.

Table 2
Multiple criteria weightings applied by three different decision makers to a supplier selection scenario.

Distinct criteria weightings applied by three decision makers				
Criteria numbers	Criteria details	Decision makers' criteria weightings		
		DM1	DM2	DM3
1	Contract bid price/material costs	0.0333	0.1000	0.1000
2	Supplier's/contractor's financial strength	0.0333	0.1000	0.0119
3	Access to finance & financial guarantees (ECAs)	0.0333	0.0200	0.1000
4	Payment schedule & credit terms	0.0333	0.0200	0.0119
5	Retention payment & warranty terms	0.0333	0.0200	0.0119
6	Track record/reputation/professionalism	0.0333	0.1000	0.0119
7	Previous global experience	0.0333	0.0200	0.1000
8	Previous local experience & procurement compliance	0.0333	0.1000	0.0119
9	Local content (workforce/suppliers)	0.0333	0.0200	0.1000
10	Community/NGO relations and CSR	0.0333	0.0200	0.0500
11	Government (local & national) relations	0.0333	0.0200	0.1000
12	Management and organizational framework	0.0333	0.0200	0.0119
13	Work force turnover and experience	0.0333	0.0200	0.0119
14	IT Software communication sophistication	0.0333	0.0200	0.0119
15	Relationships/procurement with subcontractors	0.0333	0.0200	0.0119
16	Logistics and supply base capabilities	0.0333	0.0200	0.0119
17	Supply chain management	0.0333	0.0200	0.0119
18	Equipment & resources availability	0.0333	0.0200	0.0119
19	Engineering and design expertise	0.0333	0.0200	0.0119
20	Access to appropriate technologies and patents	0.0333	0.0200	0.0119
21	Innovation/problem solving/flexibility	0.0333	0.0200	0.0119
22	Risk management & contingency planning	0.0333	0.0200	0.0119
23	Willingness to share risk	0.0333	0.0200	0.0119
24	Contract risk for sponsor	0.0333	0.0200	0.0119
25	Time overrun risk	0.0333	0.0200	0.0119
26	Cost overrun risk	0.0333	0.0200	0.0119
27	Safety, compliance & security record	0.0333	0.0200	0.0500
28	Environmental record	0.0333	0.0200	0.0500
29	Quality control and standards	0.0333	0.0200	0.1000
30	Operational plant reliability/handover expectation	0.0333	0.1000	0.0119
	Totals:	**1.0000**	**1.0000**	**1.0000**

Notes to Table 2: (1) the weightings applied sum to one when all thirty criteria are considered in each case; (2) Priority criteria for decision makers DM2 and DM3 are highlighted with a gray background.

Table 3
Simple linguistic variables used to initially assess each criterion for each bidder, and numerical equivalents assigned to those linguistic assessments.

Conversion of linguistic assessments into numerical equivalents									
Linguistic terms use for assessment	Symbols applied	Assigned linear score	Assigned nonlinear score	Triangular fuzzy set			Intuitionistic fuzzy Set(IFS)		
				Low	Central	High	μ	ν	π
Very poor	VP	1	1	0.00	0.00	0.25	0.20	0.70	0.10
Poor	P	2	2	0.00	0.25	0.50	0.30	0.60	0.10
Moderate	M	3	5	0.25	0.50	0.75	0.45	0.35	0.20
Good	G	4	8	0.50	0.75	1.00	0.60	0.30	0.10
Very good	VG	5	10	0.75	1.00	1.00	0.80	0.10	0.10

Note to Table 3: The numerical equivalents assigned to the linguistic assessments are appropriate for the MCDM methodologies applied and compared in this study, but other scores could be applied.

The simple linguistic system provides a good basis for initial qualitative assessment. It also means that the criteria for which a higher outcomes means a better performance can be assessed using the same linguistic variable scale as the criteria for which a lower outcome means a better performance (i.e., criteria 1, 13, 24, 25 and 26). For example, an assessment of "Poor" can mean a high-expected outcome for criteria 13, or a low-expected outcome for criteria 6. Table 4 details the linguistic assessment of each bid for each criterion applying the linguistic scale defined in Table 3.

The MCDM scenario can be expressed in matrix format as follows:

$$D = \begin{array}{cc} & \begin{array}{cccc} A_1 & A_2 & \ldots & A_n \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ \vdots \\ C_m \end{array} & \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1n} \\ x_{21} & x_{22} & \ldots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \ldots & x_{mn} \end{bmatrix} \end{array} \tag{1}$$

\raster="fx2"\fleqno \hbox{(1)}
where D is a decision matrix consisting of:

$A = \{A_1, A_2, \ldots A_n\}$ a set of n alternative bidders,

Table 4
Linguistic assessment of supplier selection scenario defined (i.e., 5 bidders – EPC1 to EPC5- and 30 criteria) using the linguistic scale defined in Table 3.

Linguistic assessment of supplier-selection criteria						
Sector	Criteria numbers	Decision makers' assessments of five EPC bidders				
		EPC1	EPC2	EPC3	EPC4	EPC5
Finance	1	M	P	G	G	VG
	2	G	M	VG	M	G
	3	M	G	M	G	P
	4	P	M	VG	VG	G
	5	P	M	VG	VP	G
Reputation	6	G	G	P	M	M
	7	M	VG	VG	P	G
	8	G	G	VP	M	M
Local focus	9	G	M	VG	VG	G
	10	M	P	G	G	VG
	11	G	M	VG	G	VP
Project management	12	G	G	P	M	M
	13	M	VG	VP	P	G
	14	VG	G	M	VG	P
	15	P	M	VG	VG	G
Supply chain	16	M	VG	M	P	G
	17	M	P	M	G	VG
	18	G	G	P	M	M
Technical/Engineering	19	G	G	P	M	M
	20	M	VG	G	P	G
	21	M	VG	VP	P	G
Risk mitigation	22	P	M	VG	VG	G
	23	M	P	VP	G	VG
	24	VG	G	M	VP	P
	25	M	P	VP	G	VG
	26	VG	G	M	VP	P
HSSE	27	VG	G	M	G	P
	28	P	M	VG	G	G
Quality/Performance	29	P	M	VP	G	VP
	30	M	P	G	VP	M

Notes to Table 4: 1) One approach is to reach consensus such that all three decision makers are in agreement concerning the linguistic assessments applied. In practice, that agreement may not always be easy to achieve, and each decision maker may have their own assessment matrix that needs to be integrated via importance weightings. 2)As introduced in Table 3 the linguistic descriptors used to assess supplier performance are abbreviated as follows: VP means "very poor"; P means "poor"; M means "moderate"; G means "good"; and, VG means "very good".

$C=\{C_1,C_2...C_m\}$ a set of m criteria to assess the bidders
$Wc=\{Wc_1,Wc_2...Wc_m\}$ a set of weights applied to the m criteria,
$Wg=\{Wg_1,Wg_2...Wg_l\}$ a set of importance weights applied to the assessments of l decision makers.

The supplier selection scenario and assessment is now defined by three main dimensions:

$n = 5$ bids/bidders to evaluate
$m = 30$ criteria with which to assess the bids
$l = 3$ decision makers conducting independent assessments of the n bidders based on the information available for the m criteria.

A fourth dimension influencing the final decision recommendation is associated with the importance weight (W_g) applied to each decision maker's assessment. Individual elements of the criteria weights set (W_c) and the importance weights set add to 1.

The decision makers in this scenario agree on the linguistic, qualitative assessments (Table 4) applied to each criteria for each bidder, but not on the criteria weighting. Hence, there is just one assessment matrix to evaluate in the case considered. In practice there may be multiple assessment matrices to evaluate, but all the methodologies described here can easily cope with that issue.

This scenario is evaluated and ranking of the bids is then compared for six MCDM methodologies, with pros and cons of each method discussed. There are several other MCDM methodologies that could also be applied to the supplier selection scenario, but the ones considered are considered sufficient to explore the scenario and highlight the issues to be addressed.

5. Simple linear and non-linear scoring methodologies using crisp numbers

These appeal to decision makers because of their simplicity and ease of calculation. It is also important to consider them for comparative purposes, in order to justify using more complex/sophisticated methodologies capable of incorporating more information. In these approaches the linguistic assessments are assigned semi-quantitative numerical scores on linear or non-linear scales (Table 3). In the linear approach used here VP = 1, P = 2, M = 3, G = 4 and VG = 5, whereas in the non-linear scales used here VP = 1, P = 2, M = 5, G = 8 and VG = 10 gives more weight to the more positive assessments. These numbers can then be expressed as matrices (Table 5), with totals of columns and rows to reveal information that helps distinguish the bidders (i.e., sums of columns) and the overall ability of all the bidders to satisfy the requirements of individual criteria (i.e., sum of rows). In matrix format sums of the columns is expressed as:

$$\tilde{A}_j = \sum_{i=1}^{m} x_{ij}; \quad with\ i = 1, \dots, m; j = 1, \dots, n \tag{2}$$

where:

$\tilde{A}_j$ is the sum of all the unweighted criteria scores for bidder j.

Of course, there a many other such scoring systems that could be defined (e.g. VP = −5, P = −2.5, M = 0, G = +2.5, VG = +5, giving equal negative weight to the poorer assessment as positive weight is given to the better assessments) depending upon the strategic objectives of the analysts/decision makers in the bid assessment process. the scale involving negative and positive numbers is not evaluated further here.

The semi-quantitative scales are arbitrarily selected. They provide a numerical basis (i.e. matrices of crisp numbers) from which to conduct mathematical calculations, but in no way represent a quantitative analysis, or provide any indications of the uncertainties associated with assessments from which they are derived.

Taken at face value, with no additional weightings applied, the sums of the columns in Table 5 can be used to rank the bidders with the linear scores suggesting a descending ranking order: EPC2 > EPC5 > EPC1 > EPC4 > EPC3; and, the non-linear scores suggesting a descending ranking order: EPC5 > EPC2 > EPC1 > EPC4 > EPC3. In the scenario described the five bidders end up with quite similar total scores. In cases, where one bidder has a much higher total score than the others using such systems, it is probably not necessary to go further and consider alternative methodologies. However, when there is little to discriminate between the bidders using such assessments then relatively small adjustments (e.g. application of weightings) and uncertainties can alter the rankings.

In addition to the sums of the columns, the sums of the rows in Table 5 also provide some insight to the assessment. Criteria 9 (local content) has the highest cumulative score (i.e., the sum of the 5-bidders scores is 21 on the linear scale and 41 on the non-linear scale), and criteria 2, 4, 7, 9, 14, 15 and 22 have higher cumulative scores than other criteria. It is possible to also discriminate the

Table 5
Linear and non-linear scoring matrices derived by converting the linguistic assessment of the supplier selection scenario defined (i.e. 5 bidders –EPC1 to EPC5 and 30 criteria) shown in Table 4 using the linear and non-linear scales defined in Table 3.

Linear and non-linear scoring matrices for supplier assessment evaluation													
Sector	Criteria numbers	Decision makers' assessments linear scores						Decision makers' assessments non-linear scores					
		EPC1	EPC2	EPC3	EPC4	EPC5	Sum linear	EPC1	EPC2	EPC3	EPC4	EPC5	Sum non-linear
Finance	1	3	2	4	4	5	**18**	5	2	8	8	10	**33**
	2	4	3	5	3	4	**19**	8	5	10	5	8	**36**
	3	3	4	3	4	2	**16**	5	8	5	8	2	**28**
	4	2	3	5	5	4	**19**	2	5	10	10	8	**35**
	5	2	3	5	1	4	**15**	2	5	10	1	8	**26**
Reputation	6	4	4	2	3	3	**16**	8	8	2	5	5	**28**
	7	3	5	5	2	4	**19**	5	10	10	2	8	**35**
	8	4	4	1	3	3	**15**	8	8	1	5	5	**27**
Local focus	9	4	3	5	5	4	**21**	8	5	10	10	8	**41**
	10	3	2	4	4	5	**18**	5	2	8	8	10	**33**
	11	4	3	5	4	1	**17**	8	5	10	8	1	**32**
Project management	12	4	4	2	3	3	**16**	8	8	2	5	5	**28**
	13	3	5	1	2	4	**15**	5	10	1	2	8	**26**
	14	5	4	3	5	2	**19**	10	8	5	10	2	**35**
	15	2	3	5	5	4	**19**	2	5	10	10	8	**35**
Supply chain	16	3	5	3	2	4	**17**	5	10	5	2	8	**30**
	17	3	2	3	4	5	**17**	5	2	5	8	10	**30**
	18	4	4	2	3	3	**16**	8	8	2	5	5	**28**
Technical/Engineering	19	4	4	2	3	3	**16**	8	8	2	5	5	**28**
	20	3	5	4	2	4	**18**	5	10	8	2	8	**33**
	21	3	5	1	2	4	**15**	5	10	1	2	8	**26**
Risk mitigation	22	2	3	5	5	4	**19**	2	5	10	10	8	**35**
	23	3	2	1	4	5	**15**	5	2	1	8	10	**26**
	24	5	4	3	1	2	**15**	10	8	5	1	2	**26**
	25	3	2	1	4	5	**15**	5	2	1	8	10	**26**
	26	5	4	3	1	2	**15**	10	8	5	1	2	**26**
HSSE	27	5	4	3	4	2	**18**	10	8	5	8	2	**33**
	28	2	3	5	4	4	**18**	2	5	10	8	8	**33**
Quality/Performance	29	2	3	1	4	1	**11**	2	5	1	8	1	**17**
	30	3	2	4	1	3	**13**	5	2	8	1	5	**21**
	Total scores:	**100**	**104**	**96**	**97**	**103**		**176**	**187**	**171**	**174**	**188**	

Note to Table 5: The criteria achieving the highest scores across all five bidders are highlighted with a gray background. The numbers in bold represent totals.

criteria with the lowest cumulative criteria scores in Table 5 (i.e.5, 8, 13, 21, 23, 24, 25, 26, 29 and 30 have cumulative linear scores of 15 or less), with criteria 29 (quality control) assessed with the lowest linear score of 11. The other criteria with intermediate scores (i.e., 16 to 18 on the linear scale) tend to have individual bidder scores dominated by linear scores of 2, 3 and 4. Such analysis reveals insight regarding which criteria are likely to be easily satisfied and others not so easily satisfied by the supplier selection decision.

Table 6 shows the adjustments made to the column-totals of the linear and non-linear matrices by applying the different weighting schemes defined for the three decision makers (from Table 2), and the importance weightings used to integrate the three-decision-

Table 6
Weighting adjustments to linear and non-linear scoring matrices for the supplier-selection scenario defined in Tables 2–5. Weighted scores are rounded to three decimal places.

Linear and non-linear decision-makers' scoring assessments weighted for criteria priorities and importance									
Linear scoring comparison		Sum of criteria weighted (W_c) scores for each decision maker			Sum of criteria weighted (W_c) & importance-weighted (W_g) scores for each decision maker				Rank (1 = best)
Bidder	Unweighted total score	DM1 ($X \times W_c$)	DM2 ($X \times W_c$)	DM3 ($X \times W_c$)	DM1 ($X \times W_c \times W_g$)	DM2 ($X \times W_c \times W_g$)	DM3 ($X \times W_c \times W_g$)	Fully weighted total assesment	
EPC1	100	3.333	3.440	3.245	1.111	1.147	1.082	3.340	4
EPC2	104	3.467	3.280	3.343	1.156	1.093	1.114	3.363	1
EPC3	96	3.200	3.200	3.626	1.067	1.067	1.209	3.342	3
EPC4	97	3.233	3.060	3.638	1.078	1.020	1.213	3.310	5
EPC5	103	3.433	3.500	3.143	1.144	1.167	1.048	3.359	2
***Non*-linear scoring comparison**		**Sum of criteria weighted (W_c) scores for each decision maker**			**Sum of criteria weighted (W_c) & importance-weighted (W_g) scores for each decision maker**				**Rank (1 = best)**
Bidder	**Unweighted total score**	**DM1 ($X \times W_c$)**	**DM2 ($X \times W_c$)**	**DM3 ($X \times W_c$)**	**DM1 ($X \times W_c \times W_g$)**	**DM2 ($X \times W_c \times W_g$)**	**DM3 ($X \times W_c \times W_g$)**	**Fully weighted total Assesment**	
EPC1	176	5.867	6.240	5.650	1.956	2.080	1.883	5.919	5
EPC2	187	6.233	5.740	5.881	2.078	1.913	1.960	5.951	4
EPC3	171	5.700	5.740	6.788	1.900	1.913	2.263	6.076	2
EPC4	174	5.800	5.400	6.862	1.933	1.800	2.287	6.021	3
EPC5	188	6.267	6.400	5.643	2.089	2.133	1.881	6.103	1

makers' scores (i.e., all 0.3333 in the base case) into a final assessment and ranking. These simple weightings adjustments are expressed in matrix format as:

$$p_{jk} = \sum_{i=1}^{m} x_{ij} \times Wc_i x Wg_k; \;\; with\; i = 1, \ldots, m;\; j = 1, \ldots n; k = 1, \ldots, l \tag{3}$$

where:

p_{jk} is the sum of all the weighted criteria scores for bidder j with importance weightings for decision maker k applied

$$\widetilde{P}_j = \sum_{k=1}^{l} p_{jk;}; \;\; with\; j = 1, \ldots, n;\; k = 1, \ldots, l \tag{4}$$
$$where: \widetilde{P}_j \; is\; the\; weight\; adjusted\; score\; for\; bidder\; j$$

Table 6 reveals that the application of the different criteria weightings of the decision makers results in a modified ranking order of bidders, with the weighted linear scores suggesting a descending ranking order: EPC2 > EPC5 > EPC3 > EPC1 > EPC4; and, the weighted non-linear scores suggesting a descending ranking order: EPC5 > EPC3 > EPC4 > EPC2 > EPC1. From Table 6 it is apparent that DM1 ranks#1 EPC2 (i.e., the same as the unweighted ranking, because DM2 applies equal weighting to all criteria), DM2 ranks#1 EPC5 and DM3 ranks#1 EPC4, but also ranks EPC3 just behind EPC4. This results in EPC3 moving significantly up the rankings in the weighted analysis in comparison to its last-place ranking in the unweighted analysis. The underlying reason for these differences in the weighted-analysis rankings versus the unweighted-analysis rankings is that more of the scores assigned to the higher-weighted criteria are higher overall for EPC3 than the other bidders (e.g., review the scores for criteria 30 – last row in Table 4 – for which EPC3 scores better than the other bidders; note that criteria 30 is one of only four criteria which DM2 weights more highly than other criteria, as identified in Table 2).

The remainder of this study focuses upon the application of more sophisticated MCDM methodologies (i.e. different TOPSIS calculations) to the supplier selection scenario described in order to establish what additional information they can reveal and how they might further help decision-makers to reach more credible rankings and selections.

6. TOPSIS methodology using crisp numbers

The clear distinction between a TOPSIS methodology and the simple linear and non-linear scoring approaches already described is that TOPSIS is taking into account the distance of each criteria assessment from both the positive ideal and negative ideal, with the relative closeness (RC) index used for ranking maximizing the distance from the negative ideal. The steps involved in conducting a TOPSIS analysis are described concisely in numerous publications (e.g. Krohling and Campanharo, 2011; Ghazanfari et al., 2014) so will not be repeated in detail here except for the key mathematical formulations required.

The decision matrix (i.e., equation (1)) requires normalization to transform it into a dimensionless matrix. Dividing the matrix elements in each row (criteria) by the square root of the sum off the squares of the elements of that row yields a normalized decision matrix with normalized value elements of r_{ij}:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}, \; with\; i = 1, \ldots, m;\; j = 1, \ldots, n \tag{5}$$

$$where\; R\; is\; the\; normalized\; decision\; matrix: \; R = \left[r_{ij}\right]_{m \times n}, \;\; with\; i = 1, \ldots, m; j = 1, \ldots, n \tag{6}$$

The normalized matrix is then adjusted by the criteria weights (W_c) to yield a weighted matrix with P_{ij} elements, by applying equation (3), but excluding the importance weighting (W_g) adjustments at this stage.

The positive-ideal solution (A^+) and the negative-ideal solution (A^-) can then be identified:

$$A^+ = \left(p_1^+, p_2^+, \ldots, p_n^+\right) \tag{7}$$

$$A^- = \left(p_1^-, p_2^-, \ldots, p_n^-\right) \tag{8}$$

where:

$$p_i^+ = \max_j \left(p_{ij}\right), \;\; with\; i = 1, \ldots, m;\; j = 1, \ldots, n \tag{9}$$

$$p_i^- = \min_j \left(p_{ij}\right), \;\; with\; i = 1, \ldots, m;\; j = 1, \ldots, n \tag{10}$$

With different scoring systems to the one used here (Tables 4 and 5) which involve absolute financial data for costs and benefits, the positive-ideal solution for benefit criteria is the maximum among the n bidders, and the positive-ideal solution for cost criteria is the minimum among the n bidders. For the negative-ideal solution the reverse is the case: the negative-ideal solution for benefit criteria is the minimum among the n bidders, and the negative-ideal solution for cost criteria is the maximum among the n bidders. For the linguistic scoring system applied here equations (9) and (10) suffice.

The Euclidian distance of each element in the normalized matrix from the positive-ideal and negative-ideal solutions for each criterion (i) is then calculated and summed for each bidder (j).

$$d_j^+ = \sqrt{\sum_{i=1}^{m} \left(p_i^+ - p_{ij}\right)^2} with\; i = 1, \ldots, m;\; j = 1, \ldots, n \tag{11}$$

$$d_j^- = \sqrt{\sum_{i=1}^{m} \left(p_{ij} - p_i^-\right)^2} with\; i = 1, \ldots, m; j = 1, \ldots, n \tag{12}$$

where:

d_j^+ is the Euclidian distance from the positive-ideal solution (A^+)
d_j^- is the Euclidian distance from the negative-ideal solution (A^-)

In the case of distinct cost and benefit scoring systems absolute Euclidian distances are calculated using equations (11) and (12) to avoid the involvement of negative numbers. The summed Euclidian distances for each alternative bidder are then used to calculate a relative closeness index (RC_j).

$$RC_j = \frac{d_j^-}{d_j^+ - d_j^-}, \; with\; j = 1, \ldots, n \tag{13}$$

The magnitude of the relative closeness index is then used to rank the alternative bidders.

The bidder with the highest relative closeness index is ranked #1 because they are, taking account of all the m criteria assessed, farthest from the negative ideal solution and therefore closest to the positive ideal solution.

Table 7 shows the results of a TOPSIS analysis based on the linear numerical scoring matrix (Table 5) for the supplier selection scenario defined in Tables 2–4 The different weighting schemes defined for the three decision makers (from Table 2), and the importance weightings used to integrate the three-decision-makers' scores (i.e., all 0.3333 in the base case) into a final assessment and ranking are applied.

The TOPSIS analysis presented here is conducted in two steps. The first step involves separate TOPSIS analysis applying the weightings of each decision maker. The second step involves applying a second TOPSIS analysis using the RCj ratios for each decision maker from the first step with the defined importance weightings applied to each. The integrated RC_g ratio calculated by the second step then incorporates both criteria and importance weightings.

Table 7 indicates that TOPSIS analysis using linear numerical scores suggests EPC3 as the rank#1 bidder based upon the highest -relative closeness ratio (RCg) taking into account the base case criteria and importance weightings. A closer inspection of the TOPSIS analysis from each decision-maker's perspective reveals rankings that differ from the combined ranking, as follows: DM1 ranking is EPC5 > EPC2 > EPC1 > EPC4 > EPC3; DM2 ranking is EPC5 > EPC1 > EPC3 > EPC2 > EPC4; and DM3 ranking is EPC4 > EPC3 > EPC2 > EPC1 > EPC5. Note that none of the decision makers independently ranks EPC3 at the top, even though the integrated analysis (applying equal importance weightings in the base case of 0.3333 to each decision maker's assessment) ranks#1 EPC3.

A simple addition of the decision maker's RC indices (i.e., columns 2 to 4 in Table 7) yields a ranking of EPC3 > EPC5 > EPC1 > EPC4 > EPC2, which selects the same rank#1 and rank#5 as the integrated importance-weighted TOPSIS analysis, but differs in the ranking order of the other bidders. A closer inspection of the final four columns of Table 7 reveals that it is, in fact, the lower Euclidean distance calculated from the positive ideal solution (i.e., Dg+) that is responsible for EPC3 achieving the highest RCg index in the TOPSIS analysis.

The three multi-criteria analysis methodologies applied so far to the defined supplier-selection scenario (i.e. weighted-linear ranking, weighted-non-linear ranking and TOPSIS) all involve crisp numbers. They do not take into account the uncertainties that are likely to be associated with the qualitative linguistic assessments from which the numerical matrices involved in the analysis are based.

7. Fuzzy TOPSIS methodology transforms crisp numerical matrix into fuzzy sets

In order to introduce an element of uncertainty into the criteria assessments for the supplier selection scenario the linguistic assessments are converted into sets of triangular numbers applying the zero to one scale illustrated in columns three to five in Table 3. Triangular fuzzy numbers offer an effective way of capturing uncertainty and subjectivity in MCDM (e.g., Kahraman et al., 2004). Each assessment is then represented by a fuzzy number expressed as a triplet of high, central and low numbers, defined here to be real numbers that belong to the fuzzy set $\tilde{a}$ [0,1]. Each triangle represents a defined set of overlapping numbers within that range with an easy-to-calculate membership function $\mu_{\tilde{a}}(x)$, as illustrated in Fig. 1.

The same TOPSIS methodology using fuzzy numbers is applied to the one described above using crisp numbers (i.e. equations (3)–(13)), with the exception that the Euclidean distances to the positive-ideal solution and negative-ideal solution use the vertex method expressed as equations (14) and (15), instead of equations (11) and (12), to extract crisp distances from the differences between the three elements of the fuzzy elements of the decision matrix and the maximum and minimums established for the fuzzy elements of each criteria.

$$d_j^+ = \sqrt{\sum_{i=1}^{m}(1/3)\left[\left(pa_i^+ - pa_{ij}\right)^2 + \left(pb_i^+ - pb_{ij}\right)^2 + \left(pc_i^+ - pc_{ij}\right)^2\right]},\quad with\ i = 1,\ldots,m;\ j = 1,\ldots,n \tag{14}$$

$$d_j^- = \sqrt{\sum_{i=1}^{m}(1/3)\left[\left(pa_{ij} - pa_i^-\right)^2 + \left(pb_{ij} - pb_i^-\right)^2 + \left(pc_{ij} - \overline{p}c_i^2\right)\right]},\quad with\ i = 1,\ldots,m;\ j = 1,\ldots,n \tag{15}$$

where:

d_j^+ is the Euclidian distance from the positive-ideal solution (A^+)
d_j^- is the Euclidian distance from the negative-ideal solution ($A-$)
a, b, c are the triplet of numbers constituting fuzzy number $\tilde{\mathbf{a}}$

The fuzzy TOPSIS analysis is conducted in two steps similar to the TOPSIS methodology. The first step involves three separate fuzzy TOPSIS analysis applying the weightings of each decision maker. The second step involves applying a basic TOPSIS analysis using the RC_j ratios (i.e., crisp numbers derived from equation (13)) for each decision maker derived from the first fuzzy TOPSIS step, with the defined importance weightings applied to the RC ratios calculated for each decision maker in step one. The integrated RCg

Table 7
Weighting adjustments to the TOPSIS analysis based upon the linear scoring matrices (Table 5) for the supplier-selection scenario defined in Tables 2–4 Weighted scores are rounded to three decimal places.

Decision-makers' non-fuzzy TOPSIS assessments weighted for criteria priorities and importance										
Non-fuzzy TOPSIS linear score	Criteria weighted (W_c) relative closeness measures for each decision maker			Sum of criteria weighted (W_c) & importance weighted (W_g) relative closeness measures for each decision maker			Euclidian distance measures for integrated analysis		Integrated TOPSIS analysis for W_c and W_g weightings	
Bidder	DM1 ($W_c - RC$)	DM2 ($W_c - RC$)	DM3 ($W_c - RC$)	DM1 (W_c & $W_g - RC$)	DM1 (W_c & $W_g - RC$)	DM1 (W_c & $W_g - RC$)	Distance from positive-ideal (Dg+)	Distance from negative-ideal (Dg+)	Relative closeness Index (RCg)	Rank based on RCg (1 = best)
EPC1	0.527	0.600	0.458	0.176	0.200	0.153	0.507	0.511	0.502	2
EPC2	0.545	0.452	0.501	0.182	0.151	0.167	0.524	0.429	0.450	5
EPC3	0.493	0.554	0.579	0.164	0.185	0.193	0.441	0.516	0.540	1
EPC4	0.499	0.400	0.655	0.166	0.133	0.218	0.522	0.523	0.500	3
EPC5	0.547	0.618	0.431	0.182	0.206	0.144	0.523	0.523	0.500	4

ratio calculated by the second step then incorporates both criteria and importance weightings.

A triangular-fuzzy-set-scoring system is applied here (Fig. 1) to the linguistic assessments of the defined supplier-selection scenario (Table 3) in evaluating the fuzzy-TOPSIS methodology. Table 8 presents the key results of the fuzzy-TOPSIS analysis using defined fuzzy-set-scoring system (Table 3, Fig. 1) applied to base case assumptions for the defined supplier selection scenario. The results suggest reasonably close agreement with the TOPSIS analysis, i.e., EPC3 as the rank#1 bidder, based upon the highest -relative closeness ratio (*RCg*) taking into account the base case decision-makers' criteria and importance weightings (i.e. subjective weightings only). However, the integrated ranking order of alternative bidders differs, i.e., EPC3 > EPC4 > EPC2 > EPC5 > EPC1; rather than EPC3 > EPC1 > EPC4 > EPC5 > EPC2 for TOPSIS (Table 7).

A closer inspection of the fuzzy TOPSIS analysis from each decision-maker's perspective reveals rankings that differ from the combined ranking, as follows: DM1 ranking is EPC2 > EPC5 > EPC1 > EPC4 > EPC3; DM2 ranking is EPC5 > EPC1 > EPC2 > EPC3 > EPC4; and DM3 ranking is EPC4 > EPC3 > EPC2 > EPC1 > EPC5. Note once again that none of the decision makers independently ranks EPC3 at the top, even though the integrated analysis (applying equal importance weightings in the base case of 0.3333 to each decision maker's assessment) ranks#1 EPC3.

A simple addition of the decision maker's RC indices (i.e., columns 2 to 4 in Table 8) yields a ranking of EPC5 > EPC2 > EPC1 > EPC3 > EPC4, with the preferred integrated selection of the fuzzy TOPSIS analysis in fourth position. A closer inspection of the final four columns of Table 8 reveals that it is the middle ranking of both Euclidean distances (i.e. Dg+ and Dg−) that work together in

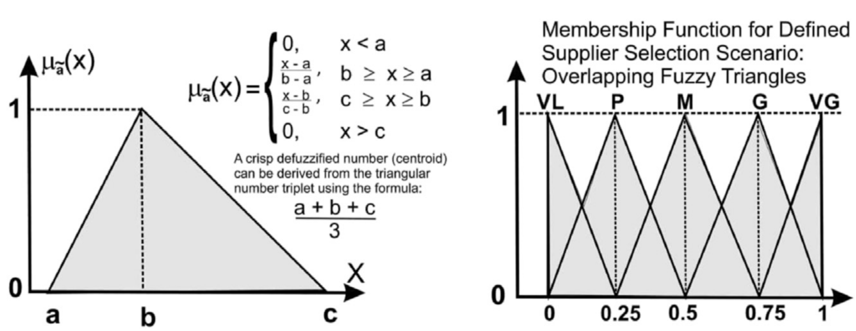

Fig. 1. Triangular fuzzy number ã defined as a triplet (a, b, c) with membership function $\mu_{\tilde{a}}(x)$ with overlapping triangular numbers belonging to fuzzy set Ã used for the defined supplier selection scenario. Diagram modified after Krohling and Campanharo (2011) and Ghazanfari et al. (2014).

Table 8
Results of non-weighted and weighted fuzzy TOPSIS analysis based upon triangular fuzzy set scoring (Table 8) for the supplier-selection scenario defined in Tables 2–4 Weighted scores are rounded to three decimal places.

Decision-makers' fuzzy TOPSIS assessments weighted for criteria priorities and importance										
Fuzzy TOPSIS linear score	Criteria weighted (W_c) relative closeness measures for each decision maker			Sum of criteria weighted (W_c) & importance weighted (W_g) relative closeness measures for each decision maker			Euclidian distance measures for integrated analysis		Integrated TOPSIS analysis for W_c and W_g weightings	
Bidder	DM1 ($W_c - RC$)	DM2 ($W_c - RC$)	DM3 ($W_c - RC$)	DM1 (W_c & $W_g - RC$)	DM1 (W_c & $W_g - RC$)	DM1 (W_c & $W_g - RC$)	Distance from positive-ideal (Dg+)	Distance from negative-ideal (Dg+)	Relative closeness index (RCg)	Rank based on RCg (1 = best)
EPC1	0.546	0.604	0.502	0.182	0.201	0.167	0.029	0.027	0.485	5
EPC2	0.591	0.543	0.534	0.197	0.181	0.178	0.026	0.025	0.493	3
EPC3	0.501	0.515	0.628	0.167	0.172	0.209	0.026	0.031	0.540	1
EPC4	0.519	0.473	0.645	0.173	0.158	0.215	0.031	0.033	0.518	2
EPC5	0.581	0.618	0.472	0.194	0.206	0.157	0.033	0.032	0.488	4

achieving the highest RCg index for EPC3 in the fuzzy TOPSIS analysis.

8. Fuzzy TOPSIS methodology with entropy weighting

By adding an entropy calculation, an objective entropy-weighting can be added to the fuzzy-TOPSIS methodology, such that the fuzzy triangular numbers are first adjusted by the entropy weighting (i.e., objective weighting) and then adjusted by the decision makers' criteria and importance rankings (i.e., subjective weighting). The fuzzy-TOPSIS-with −entropy methodology applied here calculates entropy and entropy weights using an adaption of the method proposed by Wang et al. (2007).

In order to calculate entropy a crisp number is extracted from the fuzzy number triplets that constitute the decision matrix using equation (16) (see also Fig. 1):

$$x_{ij} = \frac{a_{ij} + b_{ij} + c_{ij}}{3}, \quad with\ i = 1, \dots, m;\ j = 1, \dots, n \tag{16}$$

This set of crisp numbers is then normalized for each criterion using equation (17):

$$r_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}}, \quad with\ i = 1, \dots, m;\ j = 1, \dots, n \tag{17}$$

The entropy value (e) for each criteria in the decision matrix element is then calculated with equation (18):

$$e_i = -k \sum_{j=1}^{n} (r_{ij} * \ln r_{ij}), \quad with\ i = 1, \dots, m;\ j = 1, \dots, n \tag{18}$$

where k is a constant, with $k = (\ln (n))^{-1}$ applied here

The set of entropy values for each criteria, *E(Ci)*, is then used to calculate the entropy weights (W_e). In order to calculate entropy weights with which to adjust the decision matrix for a TOPSIS calculation a *degree of difference* is derived by subtracting entropy e_i from one:

$$d_i = 1 - E(C_i), \quad i = 1, 2, \dots, m \tag{19}$$

The degree of difference expresses the inherent contrast intensity among the assessments of each criterion (Wang et al., 2007). The greater the relative value of d_i the more important that criteria is in discriminating between the n alternatives, and the greater weight it is objectively assigned in the calculation.

The entropy weight w for each criterion i is then calculated to form the set of entropy weights (W_e):

$$w_i = d_i \Big/ \sum_{i=1}^{m} d_i, \quad i = 1, 2, \dots, m \tag{20}$$

forming the set of entropy weights

$$W_e = (w_1, w_2, \dots w_i, w_m)$$

$$where \quad w_i \geq 0, \ \sum_{i=1}^{m} w_i = 1$$

The decision matrix of fuzzy numbers (i.e. triplets) is then adjusted by the calculated entropy (objective) weights and the decision maker's (subjective) criteria weights. The calculations then proceeds as for the fuzzy TOPSIS methodology. An entropy weight calculated in a similar way using crisp numbers could be applied to the linear, non-linear and non-fuzzy TOPSIS decision matrix, but that is not performed in this study.

Table 9 presents the key results of the fuzzy-TOPSIS with entropy analysis using defined fuzzy-set-scoring system (Table 3, Fig. 1) applied to base case assumptions for the defined supplier selection scenario. The results are distinctly different from the TOPSIS (Table 7) and fuzzy TOPSIS (Table 8) analysis, i.e., EPC2 as the rank#1 bidder, based upon the highest -relative closeness ratio (RCg) taking into account the entropy weightings (i.e., objective/calculated) plus the base case decision-makers' criteria and importance weightings (i.e. subjective weightings). The integrated ranking order of alternative bidders for this methodology and base case assumptions for the supplier-selection scenario is: EPC2 > EPC1 > EPC4 > EPC5 > EPC3; note that the alternatives ranked #1 and #5 by the TOPSIS calculation (Table 7) switch positions according to this methodology.

A closer inspection of the fuzzy TOPSIS analysis from each decision-maker's perspective reveals rankings that differ from the combined ranking, as follows: DM1 ranking is EPC2 > EPC5 > EPC1 > EPC4 > EPC3; DM2 ranking is EPC1 > EPC5 > EPC2 > EPC3 > EPC4; and DM3 ranking is EPC4 > EPC2 > EPC1 > EPC3 > EPC5. Only decision maker DM1 independently ranks EPC2 at the top, even though the integrated analysis (applying equal importance weightings in the base case of 0.3333 to each decision maker's assessment).

A simple addition of the decision maker's RC indices (i.e., columns 2 to 4 in Table 9) yields a ranking of EPC2 > EPC1 > EPC4 > EPC5 > EPC3, with the preferred integrated selection of the fuzzy TOPSIS analysis in fourth position. A closer inspection of the final four columns of Table 9 reveals that EPC2 achieves RCg index rank#1 mainly due to its low Euclidean distance from the positive

Table 9
Results of non-weighted and weighted fuzzy TOPSIS with entropy analysis based upon triangular fuzzy set scoring (Table 3, Fig. 1) for the supplier-selection scenario defined in Tables 2–4 Weighted scores are rounded to three decimal places.

Decision-makers' Fuzzy TOPSIS assessments weighted for entropy, criteria priorities and importance										
Fuzzy TOPSIS entropy weighted	Entropy (We) & criteria weighted (W_c) relative closeness measures for each decision maker			Sum of entropy (W_e), criteria weighted (W_c) & importance weighted (W_g) relative closeness measures for each decision maker			Euclidian distance measures for integrated analysis		Integrated TOPSIS analysis for W_c and W_g weightings	
Bidder	DM1 (W_c − RC)	DM2 (W_c − RC)	DM3 (W_c − RC)	DM1 (W_c & W_g − RC)	DM1 (W_c & W_g − RC)	DM1 (W_c & W_g − RC)	Distance from positive-ideal (Dg+)	Distance from negative-ideal (Dg+)	Relative closeness index (RCg)	Rank based on RCg (1 = best)
EPC1	0.555	0.613	0.494	0.185	0.204	0.165	0.033	0.048	0.589	2
EPC2	0.609	0.582	0.578	0.203	0.194	0.193	0.017	0.060	0.782	1
EPC3	0.423	0.459	0.492	0.141	0.153	0.164	0.057	0.023	0.286	5
EPC4	0.473	0.441	0.659	0.158	0.147	0.220	0.042	0.055	0.568	3
EPC5	0.569	0.603	0.375	0.190	0.201	0.125	0.055	0.042	0.432	4

ideal solution (i.e. Dg^{+}) in the fuzzy TOPSIS with entropy analysis.

In sensitivity analysis to be presented below it becomes clear that fuzzy TOPSIS and TOPSIS methodologies do not always agree on the rank#1 bidder from the supplier selection scenario defined, although that happens to be the case for base case importance weightings (i.e., as shown in Tables 7–9).

9. Intuitionistic fuzzy TOPSIS (IFT) methodologies involving entropy weightings

Intuitionistic fuzzy sets (IFS) are defined by the degree of membership (μ), the degree of non-membership (ν) and an intuitionistic index π of their elements to the IFS. Such expressions are able to effectively characterize vagueness and hesitancy associated with imprecise knowledge or information constituting the linguistic assessments of the criteria.

Atanassov (1999) defined an IFS *A* in the universe of discourse *X* with the form:

$$A = \{[x, \mu_A(x), \upsilon_A(x)]/x \in X\} \tag{21}$$

where:

$$\mu_A : X \to [0,1],\ \upsilon_A : X \to [0,1] \tag{22}$$

with the conditions that:

$$0 \le \mu_A(x) + \upsilon_A(x) \le 1,\ \forall x \in X \tag{23}$$

A, therefore, only becomes set of crisp numbers if either μ_A or ν_A equal 0 or 1. Hung and Chen (2009) provide a more complete description of IFS relationships. As $\mu_A + \nu_A$ is typically less than 1 the value of πA, the intuitionistic index of *x* in *A*, is the important fuzziness characteristic of an IFS and it is defined as:

$$\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x) \tag{24}$$

πA is also frequently referred to as the degree of hesitancy of *x* to *A*. It expresses the degree of uncertainty in the assessment as to whether x is, or is not, a member of IFS *A*.

The calculation of entropy for IFT is a key part of applying them in IFS TOPSIS methodologies. As mentioned in the introduction in an intuitionistic environment the entropy measure reveals information about the relative value content associated with each criterion, and the lower the calculated entropy for a criterion, the lower weight that should be applied to that criterion. There are several distinct methods proposed and applied for calculating IFS entropy (E^{IFS}), as mentioned in the introduction. Three such methods are applied here to evaluate and compare the defined supplier selection scenario.

The first method (termed IFT-1 here) uses the entropy computation proposed by Vlachos and Sergiadis, 2007 based upon the concepts of De Luca and Termini (1972) as applied by Hung and Chen (2009).

$$E(C_i) = \frac{-1}{n \ln 2} \sum_{j=1}^{n} [\mu_{ij}(C_i) \ln \mu_{ij}(C_i) + \upsilon_{ij}(C_i) \ln \upsilon_{ij}(C_i) - (1 - \pi_{ij}(C_i)) \ln(1 - \pi_{ij}(C_i)) - \pi_{ij}(C_i) \ln 2] \tag{25}$$

where i = 1, 2, …,m; j = 1, 2, …,n

the constant 1/(n ln2) ensures $0 \le E(C_i) \le 1$

In order to calculate entropy weights W_e with which to adjust the decision matrix for a TOPSIS calculation (i.e. in place of normalization adjustment equation (5) above) a *degree of difference is* derived by subtracting entropy from one applying equation (19), and entropy weights calculated using equation (20).

Equation (5) in the TOPSIS analysis is replaced with an entropy weighted intuitionistic fuzzy decision matrix Z can be obtained by aggregating the entropy weight vector W_e and the intuitionistic fuzzy decision matrix D as:

$$Z = W_e \otimes D = [\tilde{x}_{ij}] \tag{26}$$

where

$$\tilde{x}_{ij} = \left[1 - (1 - \mu_{ij})^{wi}, \upsilon_{ij}^{wi}\right],\ i = 1, 2, ..., m;\ j = 1, 2, ..., n \tag{27}$$

Equation (27) for weighting IFS was proposed by Atanassov (1999).

By replacing the normalized matrix derived by equation (6) with the intuitionistic fuzzy decision matrix derived by equation (27), the same TOPSIS methodology using IFS numbers is applied to the one described above using crisp numbers (i.e. equations (3)–(13)), with the exception that the Euclidean distances to the positive-ideal solution and negative-ideal solution use expressions in the form of equations (14) and (15), instead of equations (11) and (12), to extract crisp distances from the differences between the three elements of the IFS elements of the decision matrix and the maximum and minimums established for the IFS elements of each criteria. The IFT-1 methodology proposed here is an adaption of the methodology proposed by Hung and Chen (2009). The decision makers' criteria weights are applied to the Euclidian distances calculated using the entropy-weighted decision matrix.

The second method (termed IFT-2 here) uses an entropy computation based upon that proposed by Szmidt & Kacprzyk (2001):

$$E(C_i) = \frac{1}{n} \sum_{j=1}^{n} \frac{\min(\mu_{ij}(C_i), \upsilon_{ij}(C_i)) + \pi_{ij}(C_i)}{\max(\mu_{ij}(C_i), \upsilon_{ij}(C_i)) + \pi_{ij}(C_i)} \tag{28}$$

where $i = 1, 2 \ldots, m;\ j = 1, 2 \ldots, n$

The IFT-2 entropy values are then normalized and an entropy weight calculated as follows:

$$h_i = \frac{E(C_i)}{\max[E(C_i)]},\ \ i = 1, 2, \ldots, m \tag{29}$$

where h_i is normalised IFS entropy for each criteria in the decision matrix and $E(C_i)$ is the et of criteria entropy values.

This equation results in the criterion/criteria with the highest entropy being assigned a normalised entropy value of 1 (and entropy weight of zero).

In order to establish a less extreme entropy scale, a scaling factor S can be introduced into the normalization equation;

$$h_i = \frac{E(C_i)}{\max[E(C_i)] + S(\max[E(C_i)])},\ \ i = 1, 2, \ldots, m \tag{30}$$

where S is assigned a small fractional value (e.g. 0.01–0.3).

This results in some weight being assigned to the criteria with the highest entropy.

For the base case the entropy scaling factor *S* is assigned a value of 0.05. A degree of difference (d_i) is derived by subtracting normalized entropy h_i from one (i.e., applying equation (19) substituting h_i for $E(C_i)$). The entropy weight for each criterion *i* is then calculated applying equation (20), in the same way as the IFT-1 method.

These weights are then applied to the IFT-2 decision matrix using equation (27). Instead of using a modified form of equations

(14) and (15) to calculate the Euclidian distances from the positive-ideal and negative-ideal solutions, as is the case for IFT-1, simple distances from maximum and minimum weighted values for each criteria are calculated using equations (31) and (32) for the IFT-2 methodology:

$$d_j^+ = \sum_{i=1}^{m} \max\left[\left(p\mu_i^+ - p\mu_{ij}\right), \left(p\nu_i^+ - p\nu_{ij}\right),\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n\right] \tag{31}$$

$$d_j^- = \sum_{i=1}^{m} \max\left[\left(p\mu_{ij} - p\mu_i^-\right), \left(p\nu_{ij} - p\nu_i^-\right),\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \tag{32}$$

where:

d_j^+ is the Euclidian distance from the positive-ideal solution (A^+)
d_j^- is the Euclidian distance from the negative-ideal solution (A^-)
μ and ν are elements of an IFS decision matrix as defined by equations (16)–(18), with p signifying that the elements are adjusted by the entropy weights.
$p\mu_i^+$ *and* $p\nu_i^+$ are maximum weighted values for criteria i of m.
$p\mu_i^-$ *and* $p\nu_i^-$ are minimum weighted values for criteria i of m

As for the IFT-1 methodology, the decision makers' criteria weights are applied to the Euclidian distances calculated from the Entropy-weighted decision matrix.

The third method (termed IFT-3 here) uses the same overall methodology as IFS-1, but replaces the entropy computation with an adaption of that proposed by Ye (2010). Others have also proposed IFS entropy formulations that involve sine or cosine manipulations of the μ and ν elements of the IFS decision matrix (e.g. Parkash et al., 2008). Equation (25) of methodology IFT-1 is replaced with equation (33):

$$E(C_i) = \frac{1}{m}\sum_{j=1}^{n}\left\{\left\{\sin\frac{\pi \times \left[1 + \mu_{ij}(C_i) - \upsilon_{ij}(C_i)\right]}{4} + \sin\frac{\pi \times \left[1 - \mu_{ij}(C_i) - \upsilon_{ij}(C_i)\right]}{4} - 1\right\} \times \frac{1}{(\sqrt{2} - 1)}\right\} \tag{33}$$

where i = 1, 2, …,m; j = 1, 2, … s,n

The constant $1/(\sqrt{2}-1)$ ensures $0 \le E(C_i) \le 1$
π here refers to the mathematical constant pi not the degree of hesitancy

The remainder of the steps in the IFT-1 methodology is then repeated in IFS-3 methodology, so the only difference between IFT-1 and IFT-3 is the substitution of equation (33) for equation (25).

The intuitionistic fuzzy TOPSIS (IFT) analysis for the defined supplier selection scenario (Tables 2–4) is conducted with all three methodologies in two steps, similar to the approach taken for the TOPSIS and fuzzy TOPSIS methodologies. The first step involves three separate IFT analysis applying the weightings of each decision maker. The second step involves applying a basic TOPSIS analysis using the RC ratios (crisp numbers) for each decision maker derived from the first IFT step, with the defined importance weightings applied to the RC ratios calculated for each decision maker in step one. The integrated RCg ratio calculated by the second step then incorporates the entropy weightings calculated by each IFT method, the criteria weightings of each decision maker and the importance weightings assigned to each decision maker's contribution in the integrated analysis.

An intuitionistic-fuzzy-set-scoring system, derived from the linguistic assessments of the defined supplier-selection scenario (Table 3), is applied in evaluating all three intuitionistic-fuzzy-TOPSIS methodologies (i.e., IFT-1; IFT-2 and IFT-3). Clearly, there is plenty of scope to vary the values of μ, ν and π that define the intuitionistic fuzzy set assigned to each linguistic assessment to suit specific circumstances (e.g. the greater the level of uncertainty in a criterion assessment, the higher the value of π should be).

Table 10 presents the key results of the IFT analysis using the defined IFS-scoring system (Table 3) applied to base case assumptions for the defined supplier selection scenario. The results for IFT-1, IFT-2 and IFT-3 are quite distinct from the results of the TOPSIS and fuzzy TOPSIS (with and without entropy) analysis. IFT-1, IFT-2 and IFT-3 all identify EPC1 as the rank#1 bidder, based upon the highest -relative closeness ratio (RCg) taking into account the base case criteria and importance weightings. The integrated ranking orders of alternative bidders are: for IFT-1 and IFT-3 – EPC1 > EPC5 > EPC4 > EPC2 > EPC3; for IFT-2 -EPC1 > EPC4 > EPC5 > EPC2 > EPC3 (Table 10). The three IFT methodologies agree that alternatives EPC 2 and EPC3 are ranked lower than the other three bidders by quite some margin.

A closer inspection of the IFT analysis from each decision-maker's perspective reveals rankings that differ from the combined ranking, as follows for IFT-1 and IFT-3: DM1 ranking is EPC5 > EPC1 > EPC4 > EPC2 > EPC3; DM2 ranking is EPC1 > EPC2 > EPC4 > EPC5 > EPC3; and DM3 ranking is EPC5 > EPC1 > EPC4 > EPC2 > EPC3. Note that only DM2 ranks EPC1 at the top, even though the integrated analysis (applying equal importance weightings in the base case of 0.3333 to each decision maker's assessment) ranks#1 EPC1, and the RCg indices are very close between these two alternatives. Individual decision-makers rankings for IFT-2 are: DM1 ranking is EPC5 > EPC1 > EPC2 > EPC4 > EPC3; DM2 ranking is EPC2 > EPC3 > EPC1 > EPC4 > EPC5; and DM3 ranking is EPC5 > EPC1 > EPC4 > EPC2 > EPC3. Note that no decision maker individually ranks EPC1 at the top, and DM2 ranks EPC1 in third place. The select of alternative EPC1 is therefore not quite so clear cut with methodology IFT-2, which becomes more apparent with sensitivity analysis described below.

A simple addition of the decision maker's RC indices (i.e., columns 2 to 4 in Table 10) yields a ranking for IFT-1 and IFT-3 of EPC1 > EPC5 > EPC4 > EPC2 > EPC3, and for IFT-2 of EPC1 > EPC4 > EPC5 > EPC2 > EPC3 with the preferred integrated selection of each methodology in first place. A closer inspection of the final four columns of Table 10 reveals that it is the low Dg + Euclidean distances, in particular, that result in the highest RCg index for EPC1 in all IFT methodologies applied to the defined supplier selection scenario. Similarly it is the low Dg- Euclidean distances, in particular, that result in the lowest RCg index for EPC3 in all IFT methodologies.

Figs. 2–4 compare the bidder rankings derived from each of the seven methodologies described and applied in this study to the base case supplier selection scenario. What is clear is that quite different bidder selections could be justified depending upon the methodology/methodologies applied. All five of the bidders potentially achieve high-ranking positions depending upon the methodology applied. This means that it is important for decision makers to clearly justify and explain their preferred selection methodology and the strategic reasons motivating their application of a certain methodology and weighting preferences.

Table 10
Results of weighted intuitionistic fuzzy TOPSIS analysis (IFT-1, IFT-2 and IFT-3 methods) based upon defined IF-assessment sets (Table 3) for the supplier-selection scenario defined in Tables 2–4 Weighted scores are rounded to three decimal places.

Decision-makers' IFT-1 methodology TOPSIS assessments weighted for criteria priorities and importance										
IFT-1 TOPSIS entropy weighted	Entropy (W_e) & criteria weighted (W_c) relative closeness measures for each decision maker			Sum of entropy (W_e), criteria weighted (W_c) & importance weighted (W_g) relative closeness measures for each decision maker			Euclidian distance measures for integrated analysis		Integrated TOPSIS analysis for W_c and W_g weightings	
Bidder	DM1 ($W_c - RC$)	DM2 ($W_c - RC$)	DM3 ($W_c - RC$)	DM1 (W_c & $W_g - RC$)	DM1 (W_c & $W_g - RC$)	DM1 (W_c & $W_g - RC$)	Distance from positive-ideal (Dg+)	Distance from negative-ideal (Dg+)	Relative closeness Index (RCg)	Rank based on RCg (1 = best)
EPC1	0.516	0.518	0.524	0.172	0.173	0.175	0.001	0.013	0.949	1
EPC2	0.507	0.516	0.504	0.169	0.172	0.168	0.005	0.009	0.652	4
EPC3	0.488	0.491	0.470	0.163	0.164	0.157	0.013	0.000	0.000	5
EPC4	0.511	0.513	0.522	0.170	0.171	0.174	0.002	0.012	0.853	3
EPC5	0.520	0.509	0.525	0.173	0.170	0.175	0.002	0.013	0.883	2
Decision-makers' IFT-2 methodology TOPSIS assessments weighted for criteria priorities and importance										
IFT-2 TOPSIS entropy weighted	**Entropy (W_e) & criteria weighted (W_c) relative closeness measures for each decision maker**			**Sum of entropy (W_e), criteria weighted (W_c) & importance weighted (W_g) relative closeness measures for each decision maker**			**Euclidian distance measures for integrated analysis**		**Integrated TOPSIS analysis for W_c and W_g weightings**	
Bidder	**DM1 ($W_c - RC$)**	**DM2 ($W_c - RC$)**	**DM3 ($W_c - RC$)**	**DM1 (W_c & $W_g - RC$)**	**DM1 (W_c & $W_g - RC$)**	**DM1 (W_c & $W_g - RC$)**	**Distance from Positive-ideal (Dg+)**	**Distance from Negative-ideal (Dg+)**	**Relative Closeness Index (RCg)**	**Rank Based on RCg (1 = best)**
EPC1	0.512	0.502	0.522	0.171	0.167	0.174	0.003	0.010	0.762	1
EPC2	0.507	0.511	0.502	0.169	0.170	0.167	0.006	0.008	0.553	4
EPC3	0.504	0.509	0.477	0.168	0.170	0.159	0.011	0.006	0.348	5
EPC4	0.505	0.498	0.518	0.168	0.166	0.173	0.004	0.009	0.668	2
EPC5	0.514	0.478	0.535	0.171	0.159	0.178	0.006	0.011	0.642	3
Decision-makers' IFT-3 methodology TOPSIS assessments weighted for criteria priorities and importance										
IFT-3 TOPSIS entropy weighted	**Entropy (W_e) & criteria weighted (W_c) relative closeness measures for each decision maker**			**Sum of entropy (W_e), criteria weighted (W_c) & importance weighted (W_g) relative closeness measures for each decision maker**			**Euclidian distance measures for integrated analysis**		**Integrated TOPSIS analysis for W_c and W_g weightings**	
Bidder	**DM1 ($W_c - RC$)**	**DM2 ($W_c - RC$)**	**DM3 ($W_c - RC$)**	**DM1 (W_c & $W_g - RC$)**	**DM1 (W_c & $W_g - RC$)**	**DM1 (W_c & $W_g - RC$)**	**Distance from positive-ideal (Dg+)**	**Distance from negative-ideal (Dg+)**	**Relative closeness index (RCg)**	**Rank based on RCg (1 = best)**
EPC1	0.516	0.517	0.523	0.172	0.172	0.174	0.001	0.012	0.948	1
EPC2	0.507	0.516	0.504	0.169	0.172	0.168	0.005	0.009	0.656	4
EPC3	0.489	0.491	0.470	0.163	0.164	0.157	0.013	0.000	0.000	5
EPC4	0.512	0.514	0.522	0.171	0.171	0.174	0.002	0.012	0.875	3
EPC5	0.519	0.508	0.524	0.173	0.169	0.175	0.002	0.012	0.878	2

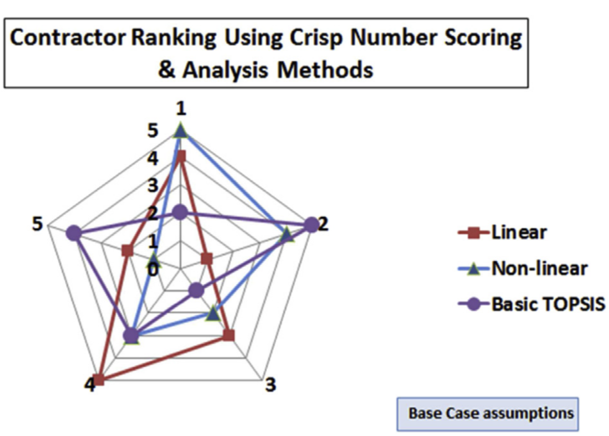

Fig. 2. Summary of base case supplier rankings of suppliers EPC1 to EPC5 (identified as 1–5, respectively, on the outer perimeter of the diagram) using three distinct crisp-number scoring, analysis and selection methodologies. The basic TOPSIS method, using the same criteria scoring system as the simple linear-scoring methodology, yields quite different bidder rankings.

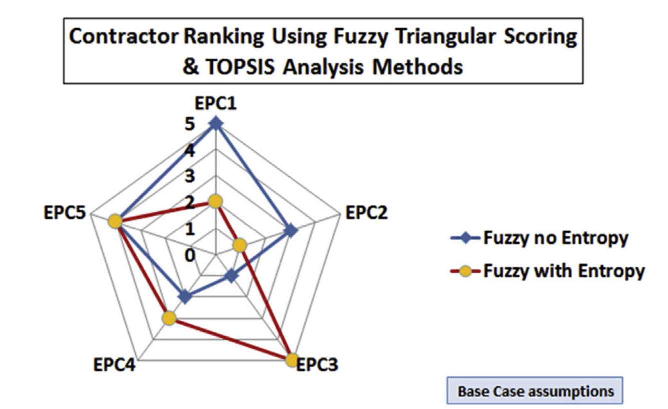

Fig. 3. Summary of base case supplier rankings of suppliers EPC1 to EPC5 (identified as 1–5, respectively, on the outer perimeter of the diagram) using a fuzzy-triangular-scoring system combined with TOPSIS analysis. These two TOPSIS methodologies, using the same fuzzy criteria scoring system as each other, yield quite different bidder rankings.

The two methodologies compared in Fig. 3 both use the same triangular-fuzzy-set scoring system derived from the linguistic variable assessments. Apart from both ranking EPC5 in rank#4

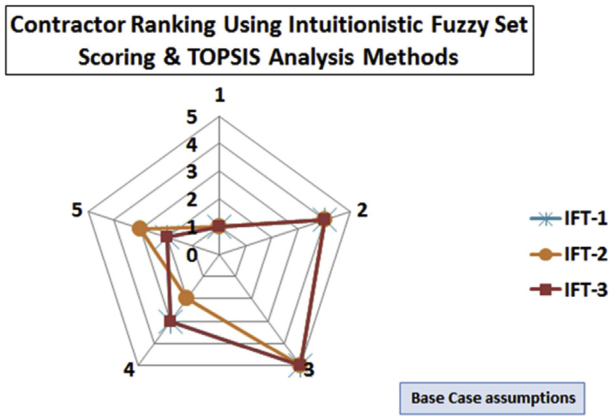

Fig. 4. Summary of base case supplier rankings of suppliers EPC1 to EPC5 (identified as 1–5, respectively, on the outer perimeter of the diagram) using an IFT-scoring system combined with TOPSIS analysis applying different entropy-weighting formulas. These three TOPSIS methodologies, using the same IFS criteria scoring system as each other, yield very consistent bidder rankings.

position they yield quite different ranking orders for the base case assumptions. Incorporating entropy weighting has a significant impact on the fuzzy triangular TOPSIS approach.

The three IFT methodologies compared in Fig. 4, all based on the same IFS scoring system (Table 3) assigned from the linguistic variable assessments, show similar bidder-selection rankings. These rankings are also quite similar to the rankings derived from the fuzzy-TOPSIS-with-entropy methodology (Fig. 3).

It is clear from Figs. 2–4 that entropy weighting, by whatever method, has a significant impact on the suggested bidder rankings derived by TOPSIS analysis and the fuzzy methodologies yield quite distinct bidder rankings compared to the simple crisp-number scoring approaches.

10. Sensitivity analysis applied to importance weightings applied to decision makers

There is scope to perform sensitivity analysis with respect to several of the base case input assumptions (e.g. subjective criteria weightings applied by the decision makers, importance weightings applied to each decision maker, numerical scoring and fuzzy set values applied to the linguistic assessments) to provide further insight to each selection methodology. When applying such methods in practice it would make sense to evaluate all such sensitivities to gain maximum insight to the factors driving the recommended selection and ranking of the alternatives being considered.

Sensitivity analysis of the importance weightings assigned to each decision maker is presented here for the defined supplier selection scenario. The importance weightings are applied in the final step of the analysis associated with each methodology, so do not fundamentally change the fundamental calculations that lead to the rankings of each decision maker in each methodology. In the base case analysis presented above the importance weightings are equal (i.e., 0.3333) for each decision maker. A change in the importance weightings applied to each decision maker means that greater or less importance is given to their particular criteria preferences/weightings listed in Table 2.

Ten sensitivity cases are evaluated (cases 2 to 11), in addition to the base case, with the bidder rankings calculated for each of the six methodologies listed in Tables 11–13. The rank#1 selection is highlighted with background shading in Tables 11–13 for each methodology applied to each case.

For the linear, basic TOPSIS and Fuzzy TOPSIS (no entropy) methodologies changing the importance weightings of the decision makers significantly changes the calculated bidder rankings from case to case (Tables 11 and 12). For the fuzzy TOPSIS methodology all five bidders are ranked #1 depending upon the case selected! With EPC1, EPC2 and EPC3 ranked #1 in three cases each, and EPC4 and EPC5 ranked #1 in one case each (Table 12). For the crisp-number-TOPSIS methodology four of the bidders are ranked #1 depending upon the case selected (Table 11), with EPC2 never being ranked higher than #3. For the linear and non-linear methodologies three of the bidders are ranked #1 depending upon the case selected (Table 11), with EPC1 and EPC4 also both achieving a rank #2 in several cases for the linear methodology. EPC1 and EPC2 also both achieving a rank #2 in several cases for the non-linear methodology (Table 11).

What stands out from the sensitivity analysis is the consistency of selection and exact replication of the IFT-1 and IFT-3 methodologies (Table 13). IFT-1 and IFT-3 select EPC1 as rank#1 in 9 of the eleven cases evaluated, and as rank#2 for the other two sensitivity cases in which DM1 is assigned a very high importance weight. IFT-2 shows less consistency in its rankings; selects EPC1 as rank#1 for four cases, but also selects EPC4 as rank#1 in four cases and EPC5 as rank#1 in three cases. None of the IFT cases select EPC2 or EPC3 as rank#1 in any cases (Table 13). The fuzzy triangular TOPSIS with entropy methodology also is consistent in its selection of EPC2 as rank#1 in 8 cases and rank#2 in the other three cases (Table 12). That methodology never selects EPC3 or EPC5 as rank#1. Three of

Table 11
Sensitivity analysis results for crisp-scoring methodologies for eleven cases varying the importance weightings (W_g) assigned to each of three decision maker (i.e., DM1 to DM3). The numbers show the ranking positions (1–5, with 1 being the best) calculated for each of five bidders (i.e., EPC1 to EPC5).

Sensitivity analysis for supplier assessment evaluation with crisp number methodologies																		
Sensitivity cases	Importance weightings (W_g)			Linear scoring					Non-linear scoring					TOPSIS (crisp linear scoring)				
	DM1	DM2	DM3	EPC1	EPC2	EPC3	EPC4	EPC5	EPC1	EPC2	EPC3	EPC4	EPC5	EPC1	EPC2	EPC3	EPC4	EPC5
Base case	0.333	0.333	0.333	4	1	3	5	2	5	4	2	3	1	2	5	1	3	4
Case 2	0.250	0.500	0.250	2	3	4	5	1	2	4	3	5	1	2	5	1	4	3
Case 3	0.500	0.250	0.250	3	1	4	5	2	5	2	3	4	1	3	5	1	4	2
Case 4	0.250	0.250	0.500	4	3	1	2	5	5	4	1	2	3	3	5	2	1	4
Case 5	0.100	0.600	0.300	2	4	3	5	1	3	5	2	4	1	2	5	1	4	3
Case 6	0.200	0.600	0.200	2	3	4	5	1	2	4	3	5	1	1	4	3	5	2
Case 7	0.600	0.200	0.200	3	1	4	5	2	5	2	4	3	1	3	5	2	4	1
Case 8	0.200	0.200	0.600	4	3	1	2	5	5	3	1	2	4	4	3	2	1	5
Case 9	0.150	0.700	0.150	2	3	4	5	1	2	4	3	5	1	2	4	3	5	1
Case 10	0.700	0.150	0.150	3	1	5	4	2	4	2	5	3	1	2	3	4	5	1
Case 11	0.150	0.150	0.700	4	3	1	2	5	5	3	2	1	4	4	3	2	1	5

Table 12
Sensitivity analysis results for fuzzy-triangular-scoring methodologies for eleven cases varying the importance weightings (W_g) assigned to each of three decision maker (i.e., DM1 to DM3). The numbers show the ranking positions (1–5, with 1 being the best) calculated for each of five bidders (i.e., EPC1 to EPC5).

Sensitivity analysis for supplier assessment evaluation with fuzzy triangular TOPSIS														
Sensitivity cases	Importance weightings (W_g)			Fuzzy TOPSIS (No entropy)					IFS fuzzy triangular with entropy					
	DM1	DM2	DM3	EPC1	EPC2	EPC3	EPC4	EPC5	EPC1	EPC2	EPC3	EPC4	EPC5
Base case	0.333	0.333	0.333	5	3	1	2	4	2	1	5	3	4
Case 2	0.250	0.500	0.250	1	3	4	5	2	2	1	5	4	3
Case 3	0.500	0.250	0.250	3	1	5	4	2	2	1	5	4	3
Case 4	0.250	0.250	0.500	4	3	1	2	5	3	1	4	2	5
Case 5	0.100	0.600	0.300	1	4	3	5	2	2	1	5	4	3
Case 6	0.200	0.600	0.200	1	3	4	5	2	2	1	5	4	3
Case 7	0.600	0.200	0.200	3	1	5	4	2	2	1	5	4	3
Case 8	0.200	0.200	0.600	4	3	1	2	5	3	2	4	1	5
Case 9	0.150	0.700	0.150	2	3	4	5	1	1	2	5	4	3
Case 10	0.700	0.150	0.150	3	1	5	4	2	2	1	5	4	3
Case 11	0.150	0.150	0.700	4	3	2	1	5	3	2	4	1	5

Table 13
Sensitivity analysis results for intuitionistic fuzzy-set-scoring methodologies for eleven cases varying the importance weightings (W_g) assigned to each of three decision maker (i.e., DM1 to DM3). The numbers show the ranking positions (1–5, with 1 being the best) calculated for each of five bidders (i.e., EPC1 to EPC5).

Sensitivity analysis for supplier assessment evaluation with intuitionistic fuzzy set TOPSIS																		
Sensitivity cases	Importance weightings (W_g)			IFT- 1					IFT- 2					IFT- 3				
	DM1	DM2	DM3	EPC1	EPC2	EPC3	EPC4	EPC5	EPC1	EPC2	EPC3	EPC4	EPC5	EPC1	EPC2	EPC3	EPC4	EPC5
Base case	0.333	0.333	0.333	1	4	5	3	2	1	4	5	2	3	1	4	5	3	2
Case 2	0.250	0.500	0.250	1	4	5	2	3	2	4	5	1	3	1	4	5	2	3
Case 3	0.500	0.250	0.250	1	4	5	3	2	2	4	5	3	1	1	4	5	3	2
Case 4	0.250	0.250	0.500	1	4	5	3	2	1	4	5	2	3	1	4	5	3	2
Case 5	0.100	0.600	0.300	1	4	5	2	3	2	3	5	1	4	1	4	5	2	3
Case 6	0.200	0.600	0.200	1	3	5	2	4	2	3	5	1	4	1	3	5	2	4
Case 7	0.600	0.200	0.200	2	4	5	3	1	2	4	5	3	1	2	4	5	3	1
Case 8	0.200	0.200	0.600	1	4	5	3	2	1	4	5	3	2	1	4	5	3	2
Case 9	0.150	0.700	0.150	1	3	5	2	4	2	3	5	1	4	1	3	5	2	4
Case 10	0.700	0.150	0.150	2	4	5	3	1	2	4	5	3	1	2	4	5	3	1
Case 11	0.150	0.150	0.700	1	4	5	3	2	1	4	5	3	2	1	4	5	3	2

the four methodologies with entropy weighting display consistency in their ranking selections across the sensitivity cases evaluated.

11. Entropy weighting scale sensitivities and other considerations

In order for entropy weighting to be effective it needs to provide a sensitive scale that clear discriminates between those criteria that are of little use in distinguishing between the alternative bidders (and apply low weightings to such criteria) from those criteria that clearly distinguish between the alternative bidders (and apply high weightings to such criteria).

Figs. 5 and 6 show that the entropy weightings applied by IFT-1 and IFT-2 are consistent. IFT-3 weightings are very similar to those applied by IFT-1.

Although these entropy scales are consistent and objectively calculated, a question that arises is how appropriate are these weightings to the objectives of specific supplier selection scenarios? Some high-entropy criteria from the list of thirty identified in Table 1 will be effectively marginalized from the selection process because entropy weights of 0.01 or less will be applied to them. Indeed if the S scaling factor is assigned a value of zero in the IFT-2 methodology the lowest entropy weight applied will be zero. This may not be appropriate if, for example, regulations or the government specify under bidding rules that certain criteria must be taken into account (e.g. local content, safety, environment, etc.). If each of the bidders is assessed similarly for those criteria, either at the poor or good end of the scoring scales, they will have high entropy and would likely be assigned a very low (or zero in some methodologies) entropy weight. Although this is objectively correct, it may be inappropriate and hard for a project sponsor to justify to the government.

In some cases it is likely to be appropriate to modify the entropy weighting scale, i.e., to introduce some flexibility/subjectivity to the scale of entropy weights applied. This is easiest to achieve using the IFT-2 methodology and changing the value of S. Table 14 shows the supplier selections for the sensitivity analysis cases applying four value of S to IFT-2 that are different from the base case value of 0.05. For the case with $S = 0$ IFT-2 selects EPC1 as rank#1 in seven out of the eleven cases. As the value of S increases EPC4 is progressively favored as rank #1, and with an S value of 0.3 IFT-2 selects EPC4 as rank#1 in all eleven cases.

Fig. 7 reveals the entropy weight scales applied by IFT-2 in the base case and four sensitivity cases (Table 14). It can be seen that the effect of increasing S is to dampen the entropy weight scale; by slightly increasing entropy weights applied to the criteria with high entropy (i.e., those that do not show much discrimination between the alternative bidders), and slightly decreasing the entropy weights applied to the key criteria with low entropy.

In many MCDM scenarios it is appropriate and desirable to have an entropy weight scale that displays maximum inverse sensitivity to entropy and applies an appropriately wide range of values. The absolute value at the centre of the entropy weighting scale should be $1/m$, so is directly related to the number of criteria involved in the analysis (e.g. Fig. 7). However, there are circumstances in which

Table 14
Sensitivity analysis results for intuitionistic fuzzy-set-scoring methodologies IFT-2, applying different entropy weight scaling values, for eleven cases varying the importance weightings (W_g) assigned to each of three decision maker (i.e., DM1 to DM3). The numbers show the ranking positions (1–5, with 1 being the best) calculated for each of five bidders (i.e., EPC1 to EPC5).

Sensitivity analysis for supplier assessment evaluation with IFT-2 varying entropy scaling factor													
Sensitivity cases	Importance weightings (W_g)			IFT-2 with S = 0.0					IFT-2 with S = 0.10				
	DM1	DM2	DM3	EPC1	EPC2	EPC3	EPC4	EPC5	EPC1	EPC2	EPC3	EPC4	EPC5
Base case	0.333	0.333	0.333	1	4	5	3	2	2	4	5	1	3
Case 2	0.250	0.500	0.250	1	4	5	2	3	2	3	5	1	4
Case 3	0.500	0.250	0.250	2	4	5	3	1	2	4	5	1	3
Case 4	0.250	0.250	0.500	1	4	5	3	2	1	4	5	2	3
Case 5	0.100	0.600	0.300	1	4	5	2	3	2	3	5	1	4
Case 6	0.200	0.600	0.200	1	3	5	2	4	2	3	5	1	4
Case 7	0.600	0.200	0.200	2	4	5	3	1	3	4	5	2	1
Case 8	0.200	0.200	0.600	1	4	5	3	2	1	4	5	2	3
Case 9	0.150	0.700	0.150	2	3	5	1	4	2	3	5	1	4
Case 10	0.700	0.150	0.150	2	4	5	3	1	3	4	5	2	1
Case 11	0.150	0.150	0.700	1	4	5	3	2	1	4	5	3	2
Sensitivity cases	**Importance weightings (W_g)**			**IFT-2 with S = 0.20**					**IFT-2 with S = 0.30**				
	DM1	**DM2**	**DM3**	EPC1	EPC2	EPC3	EPC4	EPC5	EPC1	EPC2	EPC3	EPC4	EPC5
Base case	0.333	0.333	0.333	2	4	5	1	3	2	4	5	1	3
Case 2	0.250	0.500	0.250	2	3	5	1	4	2	3	5	1	4
Case 3	0.500	0.250	0.250	2	4	5	1	3	2	4	5	1	3
Case 4	0.250	0.250	0.500	2	4	5	1	3	2	4	5	1	3
Case 5	0.100	0.600	0.300	2	3	5	1	4	2	3	5	1	4
Case 6	0.200	0.600	0.200	2	3	5	1	4	2	3	5	1	4
Case 7	0.600	0.200	0.200	3	4	5	1	2	2	4	5	1	3
Case 8	0.200	0.200	0.600	2	4	5	1	3	2	4	5	1	3
Case 9	0.150	0.700	0.150	2	3	5	1	4	3	2	5	1	4
Case 10	0.700	0.150	0.150	3	4	5	2	1	3	4	5	1	2
Case 11	0.150	0.150	0.700	1	4	5	2	3	2	4	5	1	3

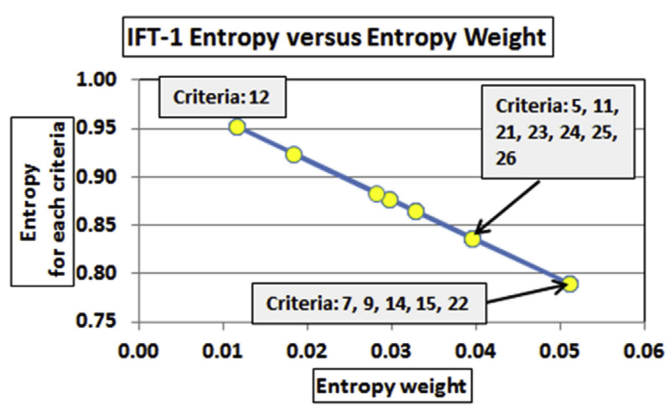

Fig. 5. Calculated entropy versus applied entropy weight for IFT-1 methodology showing the criteria for the defined supply selection analysis (Table 1) to which high weightings are applied and those (e.g. criteria 12) to which low weightings are applied.

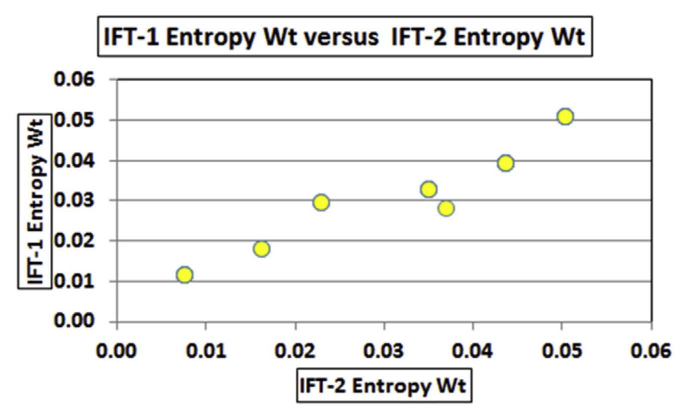

Fig. 6. Applied entropy weights for IFT-1 methodology versus IFT-2 methodology (S = 0.05; base case assumption) showing that the entropy weightings calculated by quite different methods are well correlated and extend over similar ranges.

the application of near-zero entropy weights are inappropriate, e.g. where there are few criteria involved and eliminating some may significantly impact the "objectively" selected decision outcome, and as mentioned where it would go against regulations/requirements to essentially disregard certain criteria. In such circumstances having the flexibility in the IFT to modify/dampen the entropy function can be a useful tool with which to tune IFS and fuzzy TOPSIS analysis.

The IFT-2 entropy/entropy weight calculation approach (i.e., equations (28) and (30)) is therefore proposed here as a new *intuitionistic-fuzzy-TOPSIS method with flexible-entropy weighting* suitable for MCDM scenarios in which an objective-entropy-derived- weighting scale needs to be tuned. Equation (30) is less mathematically elegant than entropy equations (25) and (33), but what it lacks in mathematical sophistication it gains in its simplicity and tuning capabilities. Modifying the S factor in equation (30) also offers some useful sensitivity analysis insight. For instance, in the supplier selection scenario employed here to illustrate various MCDM methodologies dampening the entropy weighting scale shifts the IFT selection from alternative EPC1 to EPC4 as the rank#1 recommended selection. Such information is useful in understanding the impact of entropy weighting on the ranking of alternatives.

For many gas and oil organizations supplier bid evaluation and supplier selection are core activities that can significantly impact project success or failure and the overall financial performance of the organization. However, the complex nature of the markets and macro-economic factors influencing project performance mean that MCDM must systematically integrate assessments of multiple financial and non-financial criteria. Whereas it is possible for

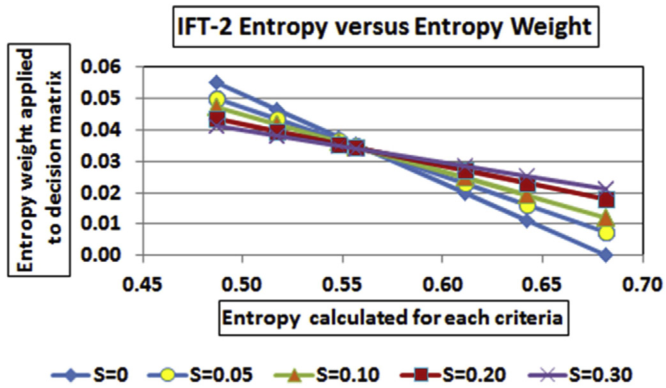

Fig. 7. Entropy versus entropy weights applied for IFT2 methodology with various values of S applied to the entropy weighting equation (30) for the defined supplier selection scenario. The point of intersection of these lines corresponds to 1/30 (0.3333…), a value of equal weighting applied to 30 criteria. The S factor has a damping effect on the entropy weight.

decision makers to place more or less weight on certain financial/non-financial criteria used in specific cases of MCDM analysis, an intuitionistic-fuzzy-TOPSIS method with flexible-entropy weighting provides an effective, repeatable and objective method to systematically incorporate the impact of a large number of assessed criteria when making a supplier-selection decision.

12. Conclusions

Supplier selection incorporating multi-criteria decision making (MCDM) is a critical and recurring activity in the gas and oil industries that has important financial and performance consequences for large facilities construction (e.g. EPC), and other project types, frequently undertaken by operators and their joint venture partners. There are a number of well-established MCDM methodologies, varying from simple to mathematically sophisticated, that are widely used across many industries and applied as tools to aid decision makers in such circumstances. However, such methodologies typically involve the influence of subjective and/or objective weightings that impose certain bias and preferences on the selections and rankings of the alternatives they consider. The impact of subjective weightings (i.e., those imposed on criteria by analysts and decision makers, and the relative importance placed on the recommendations of each decision maker in the final integrated analysis) and/or objective weightings (i.e. entropy-related calculations derived mathematically from the scoring systems applied) need to be understood and taken into account by decision makers in justifying the methodologies they rely on in making a final selection.

The joint-venture preferences for large gas and oil facilities projects and the multi-divisional structure of large gas and oil organizations frequently require that decisions consider and integrate the subjective preferences of multiple parties assessing bids, submitted by multiple bidders. This requires useful MCDM methodologies to demonstrate the flexibility to ideally include a range of objective entropy weightings (W_e), derived systematically from the scoring system applied in the analysis, combined with criteria weightings (W_c) and importance weightings (W_g) applied to each party/decision maker involved in the process.

Comparisons of the eight methodologies (linear, non-linear, TOPSIS, fuzzy TOPSIS (with and without entropy weighting), and three IFT methods for intuitionistic fuzzy TOPSIS -IFT – each involving a different entropy weighting calculation) applied to the defined supplier selection scenario highlight that quite distinct bidder rankings are calculated by each method. This observation suggests that decision makers should be prepared to compare the results of several methodologies, and run extensive sensitivities on the assumptions and assessments, before selecting a preferred bidder.

The first three methodologies use only crisp numbers and do not take into account any uncertainties associated with the linguistic assessments from which they are derived. This makes their validity questionable in scenarios where high degrees of uncertainty are known to exist (e.g. building a facility in a country lacking infrastructure where significant gas or oil developments have not taken place before). The methodologies that translate the linguistic assessments into fuzzy sets enable uncertainty to be incorporated in the calculated ranking. The fuzzy TOPSIS methodology incorporating entropy weightings and the IFT methods, provide more consistent rankings across multiple case sensitivity analysis than those methods that do not involve entropy weighting. However, the different approaches applied in the calculation of the entropy weightings in IFT methods can lead to different bidder rankings depending upon the sensitivity of the entropy weighting scale applied. It is therefore important for the decision makers to understand to which criteria the entropy weights are giving preference, and that those preferences are consistent with their decision making strategies and procurement regulations. A case is made in this study for applying a new *intuitionistic-fuzzy-TOPSIS-method-with-flexible-entropy-weighting* methodology to MCDM scenarios that can benefit from tuning the entropy weighting scale (e.g., to establish maximum and minimum thresholds for weights to be applied, to avoid near-zero weights being applied to low ranking criteria, and to gain insight from sensitivity analysis achieved by varying the entropy-weight scale.

Nomenclature

A	set of n alternative bidders
A_j	sum of unweighted criteria scores x for each of n alternatives (bidders)
$\tilde{A}^+$	set of positive ideal solutions for each of n alternatives
A^-	set of positive ideal solutions for each of n alternatives
C	set of m alternative criteria with which to assess alternatives
d_j^-	Euclidian distance from negative ideal solution for each of n alternatives
d_j^+	Euclidian distance from positive ideal solution for each of n alternatives
d_i	degree of difference (i.e. 1 minus entropy)
e_i	entropy calculated for each of m criteria in the decision matrix
E	set of entropy values e for m criteria
h_i	normalized entropy for each of m criteria
k	constant in some entropy equations
μ_A	degree of membership of intuitionistic fuzzy set A
$\mu_{\tilde{a}}$	membership function of triangular fuzzy number $\tilde{a}$
ν_A	degree of non-membership of intuitionistic fuzzy set A
p	weight-adjusted scores for m criteria for set of n alternatives
p^+	maximum of weight-adjusted criteria for set of n alternatives
p^-	maximum of weight-adjusted criteria for set of n alternatives
π_A	intuitionistic index (degree of hesitancy) of intuitionistic fuzzy set A
π	mathematical constant pi used only in equation (33)
r	normalized m criteria scores for each of n alternatives

R	normalized decision matrix (m criteria; n alternatives)
RCj	relative closeness index for each of n alternatives for each decision maker
RCg	integrated relative closeness index for each of n alternatives with Wg weights applied
W_c	criteria weights (subjective) applied by each decision maker
W_e	entropy weights (objective) to apply to each criteria
W_g	importance weights applied (subjective) to each decision makers' assessments
x	unweighted m criteria scores for n alternatives forming decision matrix D
$\bar{x}$	entropy-weighted m criteria scores for n alternatives forming IFS decision matrix D
Z	weighted intuitionistic fuzzy decision matrix

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