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Exponential utility functions aid upstream decision making

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ABSTRACT

Although the concepts and mathematics of utility theory and its application to adjusting valuations to reflect the perspectives of decision makers with a range of risk preferences have been established for decades, these concepts and numerical applications remain relatively rarely applied by decision makers in the upstream gas and oil industries. Utility functions are now extensively used to assist evaluation of oil and gas hedging and trading of financial and physical commodities from the risk preferences of the parties involved. This study makes the case for more extensive use of utility functions in the upstream gas and oil sectors by presenting cases that highlight both the conceptual and valuation benefits that result from their application.

Exponential utility functions adequately describe the risk preferences of risk-averse and risk-prone decision makers for a wide range of upstream gas and oil asset types and circumstances. Simple equations for the calculation of utility factors and expected utility factors, i.e., taking into account probabilities of a range of outcomes being realised, are presented and compared with the equivalent linear utility functions of a risk-neutral investor valuing assets based on unrisked discounted cash flow (i.e. net present value, NPV) and risked discounted cash flow (i.e., expected monetary value, EMV). The additional insight gained from applying utility functions is considered with examples for high-uncertainty exploration assets, decision makers constrained by various loss tolerances and selection of optimum gas field development plans from a number of distinct alternative plans. In all cases considered the utility functions provide decision makers with greater insight than just the consideration of NPV and/or EMV. A case is therefore made to justify more extensive use of utility functions by upstream decision makers.

1. Introduction

Since the 1960s it has been established that investors' perceptions and attitudes towards uncertainty and risk can influence the way in which they value assets and make investment decisions upon them (e.g., Hammond, 1967; Swalm, 1966). This work built upon the earlier mathematical development of classical utility theory (Von Neumann and Morgenstern, 1944; Herstein and Milnor, 1953) that itself evolved from applying game theory to economic behaviour. The mathematical definition of various utility models and preference theory has continued to evolve (e.g. Hammond, 1974; Shepherdson, 1980; Starmer, 2000; Aliev et al., 2016).

Utility theory and quantifying risk preferences with respect to oil and gas exploration and production have been widely discussed in the general context of risk analysis approaches (e.g. Macmillan, 2000; Motta et al., 2000; Ozdogan, 2004; Suslick and Schiozer., 2004; Byrska-Rąpała, 2012). Studies focused on making decisions in the upstream oil and gas industry under conditions of uncertainty, characterized by vague and imprecise estimates of reserves and future production have also touched upon the concepts of risk preferences and utility theory (e.g., Bickel and Bratvold, 2008; Bratvold and Begg, 2008). Multi-attribute utility theory is also being applied in the decision analysis associated with the decommissioning of offshore oil and gas platform (Henrion et al., 2015). However, there remains considerable scope to expand the upstream applications of these tools.

Several non-linear definitions of utility theory are available and generally match observed behaviours of investors more realistically than linear models. Prospect theory, a non-expected utility theory (Kahneman and Tversky, 1979), which allows preferences for risky decisions to be nonlinear in both outcomes and probabilities, may be a relevant approach in reflecting public perception of assigning

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high utility to very low probability events such as certain rare but severe industrial accidents and severe environmental damage (i.e., spills and pollution) (Bartczak et al., 2015). Ignoring probability weightings can lead to the expected utility of certain decision makers being under-estimated in classic utility models (Riddel, 2012).

Consider an expected utility-maximizing decision maker who has the opportunity to hold interests in two assets and wishes to rank its preference to invest in one or other of the assets. Utility theory suggests that a better decision will be made if the assets are ranked in accordance with an objective to maximize their expected utility rather than in accordance with maximizing the expected discounted cash flow value, i.e., risk adjusted net present value NPV, and/or internal rates of return, IRR.

Utility-based decision support models are now quite extensively proposed and applied in oil and gas trading and hedging activities (e.g., Cotter and Hanly, 2012; Lean et al., 2015), but less so in the upstream sector. It is worth considering how relatively-easy-to-construct expected utility models can provide insight and assistance to upstream decision making for relatively little additional effort to classic risk-adjusted discounted cash flow analysis.

We examine how risk preferences and loss aversion affect decision makers' choices significantly across the various sectors of the upstream gas and oil industries, yet the industry often doggedly relies upon unrisked discounted cash flow analysis valuations (e.g. NPV and IRR), without attempting to translate such values into expected utilities to help further refine and rationalize their decisions. This study, therefore, makes a case for the inclusion of expected utility calculations to support investment decisions in the upstream gas and oil industry.

2. Basic concepts help to visualize risk preferences

An exponential utility function for NPV (net present value – a discounted cash flow value) is useful for explaining the risk preferences of oil and gas investment decision makers that are not indifferent to their risk exposure. One way to express an exponential utility function is to use the equation:

$$U(x) = \left[1 - e^{(-r*x)}\right] \Big/ 1 - e^{(-r)} \quad (r \neq 0) \tag{1}$$

Where, U(x) is the utility function between zero and 1 for NPV x also scaled/normalised to a zero to 1 scale; and r is a risk aversion factor $\neq 0$. This equation can also be expressed using the inverse of r, i.e. a risk tolerance factor, c, such that $c = 1/r$.

In Equation (1) as the value of r increases to more positive values the curved utility function become more convex (see Fig. 1; reflecting more risk-averse tendencies), whereas, as values of r decrease to more negative numbers the curved utility function become more concave (see Fig. 1; reflecting more risk-seeking tendencies).

If $r = 0$, implying that a decision-maker is indifferent to risk Equation (1) does not apply and the utility function of that risk neutral investor is depicted by the simple relationship:

$$U(x) = x \quad (r = 0) \tag{2}$$

Equations (1) and (2) are evaluated in Table 1 and Fig. 1 to illustrate in simple terms the utility functions of risk-averse, risk-prone and risk-neutral decision makers.

Fig. 1 describes three different types of risk reaction (tolerance) behaviour; each with an objective of making decisions that maximize value as it is perceived:

Risk neutral (r=0): a linear relationship with value (i.e., Equation (2)) indicating that the decision maker is ambivalent to risk and focused on value. The appeal of a certain asset increases linearly to such an investor based upon its net present value (NPV). The slope and intercept of the straight line could be adjusted by coefficients added to Equation (2), but the essential feature is that value and utility are related in a linear manner.

Risk averse (r is positive): the curves calculated from Equation (1) are convex in shape when viewed from the top left of Fig. 1; as r increases the risk-averse utility curves become more convex. Risk-

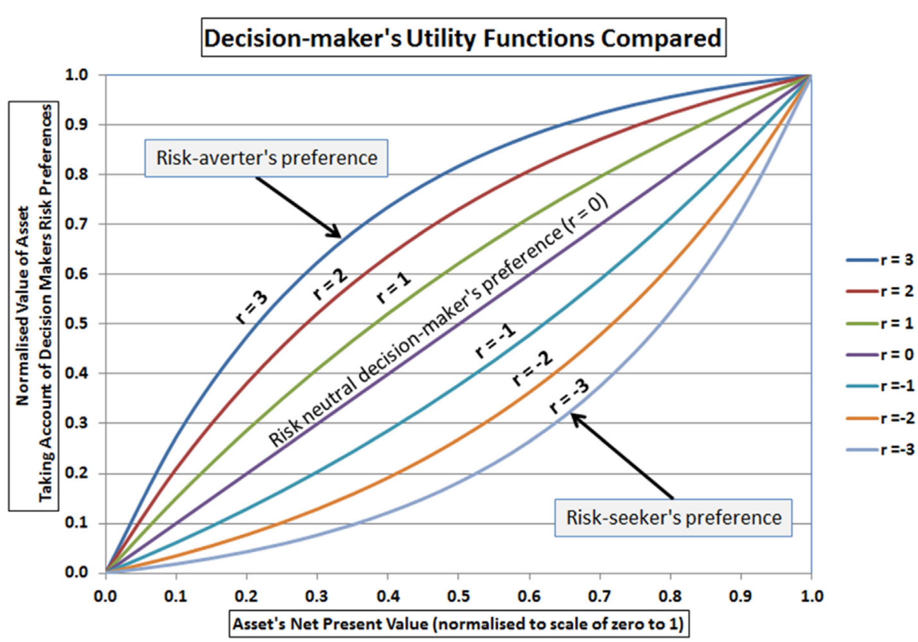

Fig. 1. Decision makers' non-linear utility function relationships to linear discounted cash flow valuations of an asset varying between zero and 1. This diagram expands upon the established concepts of exponential utility functions (Hammond, 1967; Guyaguler and Horne, 2004). The curves display the data included in Table 1 and are derived from Equations (1) And (2).

Table 1
Calculating utility function of NPV normalised to a scale of zero to 1 using a non-linear Equation (1) and a risk aversion factor (r) which is the inverse of a risk tolerance factor (c) such that r = 1/c. The risk-neutral situation (r = 0) is evaluated as a simple linear relationship between utility factor and NPV (Equation (2)).

		Net present value normalised to scale of zero to 1										
	Risk neutral:	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Risk aversion factor r	3	0.00	0.27	0.47	0.62	0.74	0.82	0.88	0.92	0.96	0.98	1.00
	2	0.00	0.21	0.38	0.52	0.64	0.73	0.81	0.87	0.92	0.97	1.00
	1	0.00	0.15	0.29	0.41	0.52	0.62	0.71	0.80	0.87	0.94	1.00
	0	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
	−1	0.00	0.06	0.13	0.20	0.29	0.38	0.48	0.59	0.71	0.85	1.00
	−2	0.00	0.03	0.08	0.13	0.19	0.27	0.36	0.48	0.62	0.79	1.00
	−3	0.00	0.02	0.04	0.08	0.12	0.18	0.26	0.38	0.53	0.73	1.00

averse decision makers apply an incremental positive risk premium to all positive values (and an incremental risk penalty to all negative values) relative to the risk neutral decision maker, which become greater as the value of r increases. Conceptually the shape of the risk-averse utility curves in Fig. 1 matches what is expected of the typical conservative decision maker, i.e. valuing small positive returns (and investments with marginal positive returns) significantly more highly than the risk neutral valuation. However, as value of an asset increases to beyond the marginal (low-value region) the relative risk premium applied by such a decision maker, relative to a risk-neutral decision maker, diminishes; hence, the convex upwards nature of risk-averse utility curves. Another way to consider the information provided by utility values from the risk-averse perspective is that in the marginal range of risk-neutral positive values, because a risk-averse investor values those assets more, such investors feel the pain more keenly for any decline in value or losses that materialise for such assets. Risk-averse decision-maker tend to strive harder (than risk-neutral or risk-seeking decision makers) to avoid incurring losses, even quite small losses, because of the severity of "pain such outcomes cause.

Risk seeking or risk prone (r is negative): the curves calculated from Equation (1) are concave in shape when viewed from the top left of Fig. 1; as r increases the risk-averse utility curves become more concave. Risk-seeking decision makers apply an incremental negative risk deduction to all positive values (and an incremental risk premium to all negative values) relative to the risk neutral decision maker, which become greater as the value of r increases. Conceptually the shape of the risk-seeking utility curves in Fig. 1 matches what is expected of the typical risk-seeking decision maker, i.e. valuing small positive returns (and investments with marginal positive returns) significantly less than the risk neutral valuation. However, as value of an asset increases to beyond the marginal (low-value region) the relative risk deduction applied by such a decision maker, relative to a risk-neutral decision maker, diminishes; hence, the concave upwards nature of risk-seeking utility curves. Another way to consider the information provided by utility values from the risk-seeking perspective is that in the marginal range of risk-neutral positive values, because a risk-seeking investor values those assets less, such investors feel the pain less keenly for any decline in value or losses that materialise for such assets. Risk-seeking decision-maker tend to cope with or rationalise losses associated with asset investments more easily, because they value them lower.

Gamblers are highly risk-prone investors and their utility function would correspond to high negative values of r. Not all decision makers fall neatly into these three categories over all ranges of value, e.g. some may display risk-averse behaviour with respect to negative values (i.e. assets that would likely involve a loss if sold at some stage over their life cycle), but risk-prone behaviour for positively valued assets. Such composite utility functions were identified and compared by Swalm (1966).

3. Expected utility versus expected monetary value

For uncertain investments, such as investing in gas and/or oil exploration prospects, the classic way of adjusting net present value (NPV) for risk is to calculate an expected monetary value (EMV) (e.g., Newendorp, 1975; Newendorp and Schulyer, 2014). EMV is a risk-adjusted discounted cash flow calculation that takes into account the time-value of money (i.e. the discount factor) and a risk factor that it typically applies in the form of a probability of success (Ps) and a probability of failure (Pf). such that

$$P_S + P_F = 1.0 \quad (3)$$

In a binary deterministic calculation this involves a possible value of success NPV outcome (Vs) being multiplied by Ps and the discounted costs associated with a possible failure (Vf) outcome multiplied by Pf. This calculation is typically made from the perspective of a risk-neutral decision maker, i.e., ignoring any risk preferences, according to Equation (4) applying the probability relationship defined in Equation (3).

$$V_S{*}P_S - V_f{*}Pf \quad (4)$$

(constrained by the rule of Equation (3)).

For example, if a decision maker has to decide upon sanctioning a risky exploration project with a 30% chance of a $4 million NPV success case and a 70% chance of a $1million loss (its share of the dry-hole drilling cost). A simple binary deterministic EMV calculation for that case would be:

$$EMV = (0.3){*}(\$4 \text{ million}) + (0.7){*}(-\$1 \text{ milllion}) = \$0.5 \text{ million}$$

The decision rule with EMV, for a risk-neutral investor, is that if the EMV is positive then the decision should be a positive one in the absence of an alternative investment with a higher NPV and if the necessary funds are available to make that investment. Of course, it would be possible to conduct a more rigorous EMV calculation on a stochastic basis to reflect a range of uncertain input assumptions (e.g., gas and oil prices, future development costs, etc.) to provide the NPV as a probability distribution. However, a stochastic analysis, usually derived through Monte Carlo simulation, typically also calculates a risked and discounted valuation from the perspective of a risk-neutral investor.

For less risky investments, where the probability of a successful outcome is high (e.g. >75% or so) and the cost of failure is zero or a very low negative figure the risked cost of failure is often approximated as zero and the risked-adjusted discounted cash flow is established by multiplying the NPV by a risk factor on a scale of zero to 1, typically in the range 0.75–1.

Table 2 and Fig. 2 illustrate how expressing EMV (or a risk-adjusted NPV) from the perspective of risk-averse and risk-seeking investors provides further insight to such investment decisions for a range of decision makers. Note that the risk-seeking-decision-makers attribute less value to the investment opportunities with lower utility factors on both sides of the zero EMV than risk-neutral or risk-averse decision makers. This is consistent with the behaviour of risk-seeking decision makers being willing, in exchange for the potential of significant gains, to accept loss outcomes more readily than more risk-averse decision makers, because such decision makers value those losses at a lower magnitude than more risk-averse decision makers.

4. Considering risk preference behaviour in relation to financial exposure limits

Decision-makers' risk preferences are likely to vary depending upon circumstances and, in particular, the availability of capital budgets.

One way to illustrate this is shown in Table 3 and Fig. 3. The utility factors in Table 3 are calculated using Equation (5):

$$U(x) = x/(x + LT) \quad (5)$$

where, U is utility factor; x is NPV expressed in monetary units; and, LT (Loss Tolerated) is the absolute value of the maximum tolerable loss in the same monetary units as x. Note that for each value of LT used a different utility scale is defined. As different absolute values of LT or financial exposure limits are used each case, each defines its own utility scale reflecting the uniqueness of each situation. It would therefore not be appropriate to express all cases in terms of a single utility scale.

Utility factors expressed in such terms will identify decision makers as risk averse when the maximum tolerable loss is a small and progressively less risk averse, as the magnitude of the maximum tolerable loss increases. The derived utility curves displayed in Table 3 and Fig. 3 reflect such decision-maker preferences.

An advantage to such an approach is that the utility scale can be varied from time to time to reflect the different circumstances in which a decision-maker is required to make decisions.

Table 4 illustrates how the utility factor scales calculated in Table 3 might be used to select between alternative investments. Both investment alternatives A and B are uncertain investments expressed deterministically to involve 5 possible identified value outcomes associated with varying probabilities. Note the probabilities of the five outcomes sum to 1.0 reflecting that the five deterministic value outcomes are deemed to account for all potential outcomes for that investment.

The left-hand three columns in Table 4 provide an EMV calculation for each investment suggesting that investment A is a better

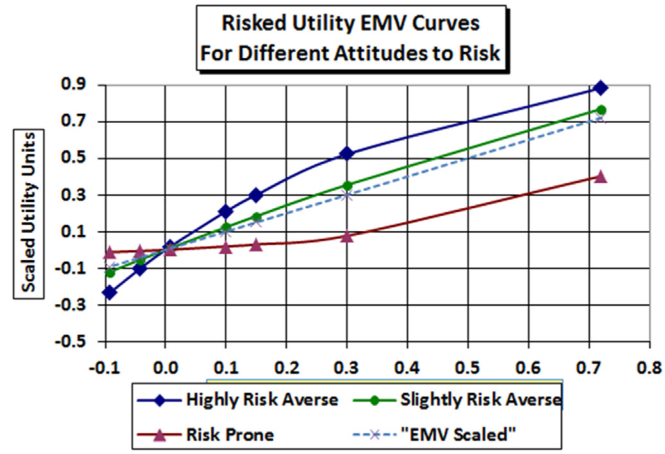

Fig. 2. Calculations from Table 2 displayed to illustrate the impact on perceived value when EMV's are adjusted to expected utility factors using a range of risk aversion coefficients. The "EMV scaled" line represents a risk neutral investor (i.e., r = 0).

Table 3
Relative utility factors calculated for a range of asset values from the perspective of maximum monetary loss that can be tolerated (Equation (5)). The asset NPVs are listed in the left-hand column. These are then adjusted to utility factors based upon the maximum tolerable losses defined in row 2 for each of the remaining columns.

Utility factors expressed in terms of magnitude of loss tolerated							
Maximum permissible loss tolerated ($ millions) →	1	5	10	15	20	25	50
Asset NPV $ millions ↓	Asset net present value expressed in utility units ($ millions)						
−5	Intolerable	Intolerable	−1.00	−0.50	−0.33	−0.25	−0.11
−4	Intolerable	−4.00	−0.67	−0.36	−0.25	−0.19	−0.09
−3	Intolerable	−1.50	−0.43	−0.25	−0.18	−0.14	−0.06
−2	Intolerable	−0.67	−0.25	−0.15	−0.11	−0.09	−0.04
−1	Intolerable	−0.25	−0.11	−0.07	−0.05	−0.04	−0.02
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.50	0.17	0.09	0.06	0.05	0.04	0.02
2	0.67	0.29	0.17	0.12	0.09	0.07	0.04
3	0.75	0.38	0.23	0.17	0.13	0.11	0.06
4	0.80	0.44	0.29	0.21	0.17	0.14	0.07
5	0.83	0.50	0.33	0.25	0.20	0.17	0.09
6	0.86	0.55	0.38	0.29	0.23	0.19	0.11
7	0.88	0.58	0.41	0.32	0.26	0.22	0.12
8	0.89	0.62	0.44	0.35	0.29	0.24	0.14
9	0.90	0.64	0.47	0.38	0.31	0.26	0.15
10	0.91	0.67	0.50	0.40	0.33	0.29	0.17

Table 2
Seven potential investments evaluated from the perspective of EMV and expected utility factors for decision makers with different risk preferences. Dividing actual expected values by the scale factor of 5 expresses utility factors in the four columns on the right side of the table on scales up to a maximum of 1.

Risked utility EMVs derived for three investors with differing attitudes to risk								
Positive investment outcome 1		Negative investment outcome 2				Highly risk averse	Slightly risk averse	Risk prone
					r:	2	0.5	−3
						Utility scale factor applied: 5		
Value of outcome ($ millions)	Probability of outcome	Value of outcome ($ millions)	Probability of outcome	Expected value ($ millions)	EMV/utility scale factor	EMV in utility units (r = 2)	EMV in utility units (r = 0.5)	EMV in utility units (r = −3)
4.00	0.9	0.00	0.1	3.600	0.720	0.883	0.768	0.402
3.00	0.5	0.00	0.5	1.500	0.300	0.522	0.354	0.076
2.50	0.5	−1.00	0.5	0.750	0.150	0.300	0.184	0.030
1.00	0.5	0.00	0.5	0.500	0.100	0.210	0.124	0.018
0.30	0.8	−1.00	0.2	0.040	0.008	0.018	0.010	0.001
0.10	0.9	−3.00	0.1	−0.210	−0.042	−0.101	−0.054	−0.006
0.20	0.7	−2.00	0.3	−0.460	−0.092	−0.234	−0.120	−0.013

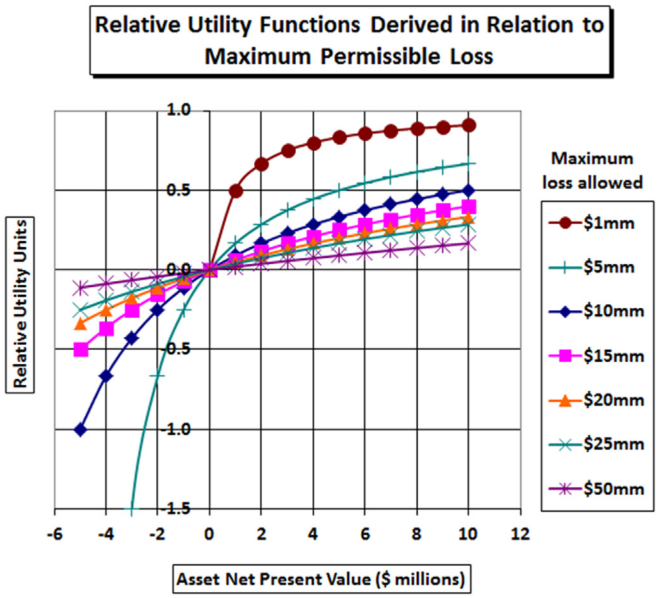

Fig. 3. Relative utility factors expressed for a range of assets valued by NPV with the utility factors determined by varying magnitudes of tolerable losses (Equation (5)). Displaying calculations listed in Table 3.

investment than investment B on a risk-adjusted discounted cash flow basis from the perspective of a risk neutral investor. On the other hand, the right-hand six columns in Table 4 adjust the NPV outcomes (column 1 of Table 4) using the utility factors from Table 3 for a loss tolerances of $ 5million, $20 million and $50 million, respectively. Comparing the utility factors for investments A and B, those derived for loss tolerances of $5 million and $20 million (i.e. more risk-averse) would select investment B in preference to investment A. However, the less-risk-averse loss tolerance of $50 million would select investment A in preference to investment B, i.e., similar to the risk-neutral EMV comparison.

In some situations it is helpful to express the utility factors for any identified level of loss tolerance on a scale of zero to 1, and to be able to invert from that utility scale back to monetary values. Equations (6)–(8) are used to facilitate this scaling and they are used here to adjust information shown in Table 3 and Fig. 3 into the information shown in Table 5 and Fig. 4 using exponential risk utility functions.

$$U(x) = m - n*e^{(-x/LT)} \tag{6}$$

where U(x) is the risk utility function for assets with a range of NPV values x; LT is the loss tolerance in the same monetary units as x and expressed in absolute terms and m and n are scaling factors to express the utility function between zero and 1 such that:

$$m = e^{(-min/LT)} \Big/ \left[e^{(-min/LT)} \Big/ e^{(-max/LT)} \right] \tag{7}$$

where, min is the minimum value of assets considered (i.e. NPV of $-5 million in Tables 3 and 5) and max is the maximum value of assets considered (i.e. NPV of $10 million in Tables 3 and 5)

$$n = 1 \Big/ \left[e^{(-min/LT)} - e^{(-max/LT)} \right] \tag{8}$$

Note that these scaling factors convert a mixture of negative and positive monetary values into a utility scale of 0–1 with curvature varied by the level of risk tolerance displayed by the decision maker.

In order to convert the utility factor scale defined by Equations (6)–(8) back into equivalent monetary terms the following relationship is used:

$$x = -LT*\ln[(m - U(x))/n] \tag{9}$$

The final row in Table 5 is calculated using Equation (9).

It is worth noting that for many upstream gas and oil investments that have a wide range of potential positive and negative outcomes it is often more useful to display utility scales with both positive and negative components (e.g., Figs. 2 and 3) rather than

Table 4
Maximum tolerable risk derived utility factors (Equation (5)) can help decision-makers to choose between investment alternatives with different budgets available to them. A decision maker with a large capital budget and tolerance for loss might adopt a risk-neutral approach and use total EMV (i.e., three left-hand columns) to select investment alternative A. On, the other hand, the same investor with a more constrained capital budget and tolerance for loss might adopt a more risk-averse approach (e.g., the middle four columns showing $5 million and $20 million loss tolerances) to select investment alternative B. Columns 4, 6 and 8 are calculated using Equation (5) with x = NPV from column 1 and LT = 5, 20 and 50 million dollars, respectively. Columns 5, 7 and 9 are calculated by multiplying columns 4, 6 and 8 by column 2 (the probability of the outcome.

EMV versus relative utility value for two risky (multiple-outcome) investments								
Uncertain investment A			More risk averse (loss tolerance $5 million)		Less risk averse (loss tolerance $20 million)		Less risk averse (loss tolerance $50 million)	
Asset NPV ($millions)	Probability of outcome	EMV ($ millions)	Outcome in relative utility units	Risked value (utility units)	Outcome in relative utility units	Risked value (utility units)	Outcome in relative utility units	Risked value (utility units)
8.0	0.2	1.60	0.62	0.123	0.29	0.057	0.14	0.028
5.0	0.2	1.00	0.50	0.100	0.20	0.040	0.09	0.018
3.0	0.2	0.60	0.38	0.075	0.13	0.026	0.06	0.011
1.0	0.1	0.10	0.17	0.017	0.05	0.005	0.02	0.002
−2.0	0.3	−0.60	−0.67	−0.200	−0.11	−0.033	−0.04	−0.013
	1.0	2.70		0.115		0.095		0.047
Uncertain investment B			More risk averse (loss tolerance $5 million)		Less risk averse (loss tolerance $20 million)		Less risk averse (loss tolerance $50 million)	
Asset NPV ($millions)	Probability of outcome	EMV ($ millions)	Outcome in relative utility units	Risked value (utility units)	Outcome in relative utility units	Risked value (utility units)	Outcome in relative utility units	Risked value (utility units)
4.0	0.2	0.80	0.44	0.089	0.17	0.033	0.07	0.015
3.0	0.3	0.90	0.38	0.113	0.13	0.039	0.06	0.017
2.0	0.2	0.40	0.29	0.057	0.09	0.018	0.04	0.008
1.0	0.1	0.10	0.17	0.017	0.05	0.005	0.02	0.002
−1.0	0.2	−0.20	−0.25	−0.050	−0.05	−0.011	−0.02	−0.004
	1.0	2.00		0.225		0.085		0.037

Table 5
Utility factors from Table 3 expressed on 0 to 1 scales based on different loss tolerances. Note that each column represents a different utility scale. The second column utility values are listed to multiple decimal places in order to show the very high utility factors above the loss tolerance level and rapidly declining utility factors of assets with values below the level of loss tolerance. For other columns the utility factors are rounded to three decimal places. The last two rows of the table show how a given utility factor in each column can be inverted to a monetary value using Equation (9).

Utility factors based on magnitude of loss tolerated expressed on 0 to 1 scale							
Loss tolerance ($ millions) (LT)	1	5	10	15	20	25	50
NPV scale maximum	10	10	10	10	10	10	10
NPV scale minimum	−5	−5	−5	−5	−5	−5	−5
Utility scale factor (m)	1.0000003059	1.052	1.287	1.582	1.895	2.216	3.858
Utility scale factor (n)	0.0067379491	0.387	0.781	1.134	1.476	1.815	3.491
Asset net present values ($ millions)	Asset net present value expressed in utility units (0–1 scale)						
−5	0	0.000	0.000	0.000	0.000	0.000	0.000
−4	0.6321207522	0.191	0.122	0.102	0.092	0.087	0.076
−3	0.8646649813	0.347	0.233	0.197	0.180	0.170	0.151
−2	0.9502132223	0.475	0.334	0.287	0.264	0.251	0.225
−1	0.9816846614	0.580	0.424	0.370	0.344	0.328	0.297
0	0.9932623568	0.665	0.506	0.448	0.419	0.402	0.367
1	0.997521553	0.735	0.581	0.522	0.491	0.473	0.436
2	0.9990884237	0.793	0.648	0.590	0.560	0.541	0.504
3	0.9996648432	0.840	0.709	0.654	0.625	0.607	0.570
4	0.9998768961	0.878	0.764	0.714	0.687	0.670	0.636
5	0.999954906	0.910	0.814	0.770	0.746	0.731	0.699
6	0.9999836042	0.936	0.859	0.822	0.802	0.789	0.762
7	0.9999941617	0.957	0.900	0.871	0.855	0.845	0.823
8	0.9999980456	0.974	0.936	0.917	0.906	0.899	0.883
9	0.9999994744	0.988	0.970	0.960	0.954	0.950	0.942
10	1	1.000	1.000	1.000	1.000	1.000	1.000
Inverting utility factors (0–1) to monetary values on the NPV scale							
Selected utility value	5.83831181877952E-006	0.420	0.141	0.713	0.313	0.269	0.496
Monetary equivalent ($ millions)	−4.9999941617	−2	−4	4	−1	−2	2

convert them to zero to 1 utility scales which make decision-maker reactions more difficult to rationalise and explain (Fig. 4).

5. Considering field development plan alternatives

Sections 2–4 address utility function applications to exploration type assets where high uncertainty can lead to outcomes varying from no positive returns on investment (i.e. drilling dry wells) to very high profitable returns on field discoveries. Utility functions can also aid decision making in field development and production assets by comparing the utility of different field development plans from the perspective of risk-averse, risk-neutral and risk-seeking decision makers.

The uncertainty in such cases is typically not whether gas and oil resources are present or whether production can be achieved, but with economic and market variables such as gas and oil prices, cost and production volumes over the production life of the asset. Some of these variables are market driven (i.e., prices and costs) others are asset driven (e.g. production volumes linked to number and location of wells drilled). Valuations of such alternative development plans therefore must integrate technical and economic uncertainties, as well as adjusting for time-value through a discounted cash flow analysis. It is typical to for gas and oil companies and their decision makers to use NPV as the primary valuation discriminator between alternative field development plans, evaluated with the aid of a series of sensitivity analysis cases and scenarios with different assumptions for the key uncertainties, both in terms of value and probability of occurrence. An additional informative set

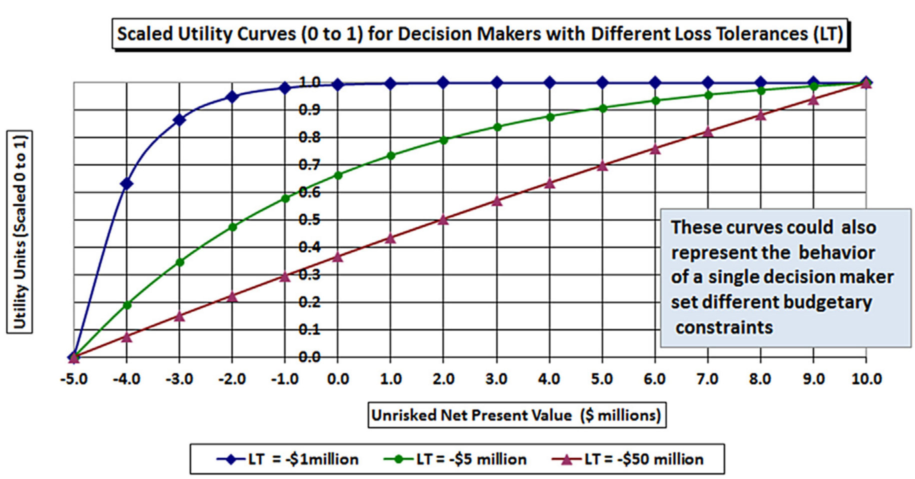

Fig. 4. Utility curves scaled 0 to 1 compared from Table 5 for different levels of loss tolerance. Limited tolerances for loss lead to more-risk-averse responses from decision makers.

Table 6
Multi-year cash flow analysis for Plan 1 of a gas field development. See Appendix 1 for details of the nine other plans and other assumptions. Base case NPV is discounted at 10%/year. Price scenarios A, B and C are listed in Table 7.

Plan 1					Scenario		
Gas field development					A	B	C
Year	Gas production (Bcf/Year)	Gas transport ($ millions)	Field opex ($ millions)	Capex ($ millions)	Cash flow ($ millions)	Cash flow ($ millions)	Cash flow ($ millions)
1	0.00	0.00	0.00	30.00	−30.00	−30.00	−30.00
2	0.00	0.00	0.00	60.00	−60.00	−60.00	−60.00
3	15.00	15.00	22.50	0.00	9.38	25.13	48.75
4	15.00	15.00	22.50	0.00	9.38	25.13	48.75
5	15.00	15.00	22.50	0.00	9.38	25.13	48.75
6	15.00	15.00	22.50	0.00	9.38	25.13	48.75
7	15.00	15.00	22.50	0.00	9.38	25.13	48.75
8	10.00	10.00	15.00	0.00	6.50	18.25	34.00
9	10.00	10.00	15.00	0.00	6.50	18.25	34.00
10	5.00	5.00	7.50	0.00	3.25	11.38	19.25
Total	100.0	100.0	150.0	NPV:	−$40.4	$22.5	$113.6
				IRR:	−7.6%	17.6%	42.7%

of sensitivity cases to run in such circumstances is to value the assets from various risk preferences/degrees of risk tolerance.

In order to conduct such analysis it is useful to modify Equations (1) And (2) to replace x (NPV) with x divided by a scaling factor, s, to expresses the utility scale in values up to about 1, but leaving the possibility of negative utility factors to apply to negative values of x. Equations (1) and (2) become Equations (10) And (11).

$$U(x) = \left[1 - e^{(-r*(x/s))}\right] \Big/ 1 - e^{(-r)} \quad (r \neq 0) \tag{10}$$

$$U(x) = x/s \quad (r = 0) \tag{11}$$

where, s is a scaling factor that is selected to be slightly above the maximum NPV value of the most positive sensitivity case assumptions. By applying scaling factor s to the NPV Equations (10) And (11) derive utility factors below 1 for all sensitivity cases calculated.

It is often appropriate to incorporate scenarios with different probabilities of occurrence for each set of technical and economic assumptions. In such cases expected utility factors can be calculated using the following relationships:

$$EU(x) = \sum (U_i * P_i) \quad (\text{with } i = 1 \text{ to } k) \tag{12}$$

Where, EU(x) is the expected utility value of NPV x, combining utility factor calculations derived from Equation (10) or 11 for each of k scenarios with the probability of occurrence P of each scenario, constrained by $\sum Pi = 1$.

To illustrate the value of utility factors in comparing alternative field development plans, integrating technical and economic input, a hypothetical gas field with ten alternative development plans is evaluated here. Each plan involves a multi-year production and cash flow analysis spread over ten years, with years 1 and 2 involving initial capital investment and field production commencing in year 3. Table 6 provides the cash flow analysis for

Table 7
Base case price scenarios A, B and C and associated probability assumptions used to evaluate the ten potential gas field development plans.

Scenario	Gas price	Probability
A	3.00	0.25
B	5.00	0.50
C	8.00	0.25
	$/mmBtu	1.000

Table 8
Expected utility value calculations for potential development Plan 1 for a range of risk tolerances using NPVs calculated in Table 6 using gas price and probability scenarios listed in Table 7.

Scale factor 400	Expected utility calculation	Plan 1		
	Scenario:	A	B	C
r	Net present value ($ millions):			
−1	Utility value	−0.056	0.034	0.191
	U*P	−0.014	0.017	0.048
	Expected utility value		0.051	
−2	Utility value	−0.029	0.019	0.120
	U*P	−0.007	0.009	0.030
	Expected utility value		0.032	
−3	Utility value	−0.014	0.010	0.070
	U*P	−0.003	0.005	0.018
	Expected utility value		0.019	
0	Utility value	−0.101	0.056	0.284
	U*P	−0.025	0.028	0.071
	Expected utility value		0.074	
1	Utility value	−0.168	0.087	0.391
	U*P	−0.042	0.043	0.098
	Expected utility value		0.099	
2	Utility value	−0.259	0.123	0.501
	U*P	−0.065	0.062	0.125
	Expected utility value		0.122	
3	Utility value	−0.373	0.163	0.603
	U*P	−0.093	0.082	0.151
	Expected utility value		0.139	

Table 9
Expected utility values for all ten potential development plans for a range of risk preferences applying the base case assumptions listed in Table 7.

Expected utility value for gas field development alternatives						
Risk preferences		Plan 1	Plan 2	Plan 3	Plan 4	Plan 5
Risk seeking	r = −3	0.019	0.018	0.018	0.019	0.022
	r = −2	0.032	0.031	0.030	0.032	0.037
	r = −1	0.051	0.050	0.049	0.052	0.058
Risk neutral	r = 0	0.074	0.073	0.072	0.078	0.082
Risk averse	r = 1	0.099	0.100	0.098	0.108	0.108
	r = 2	0.122	0.124	0.123	0.138	0.131
	r = 3	0.139	0.144	0.143	0.164	0.146
		Plan 6	Plan 7	Plan 8	Plan 9	Plan 10
Risk seeking	r = −3	0.017	0.019	0.020	0.022	0.024
	r = −2	0.029	0.031	0.034	0.036	0.039
	r = −1	0.045	0.049	0.054	0.056	0.061
Risk neutral	r = 0	0.065	0.072	0.079	0.080	0.089
Risk averse	r = 1	0.085	0.095	0.106	0.105	0.118
	r = 2	0.103	0.116	0.132	0.126	0.144
	r = 3	0.113	0.130	0.153	0.139	0.163

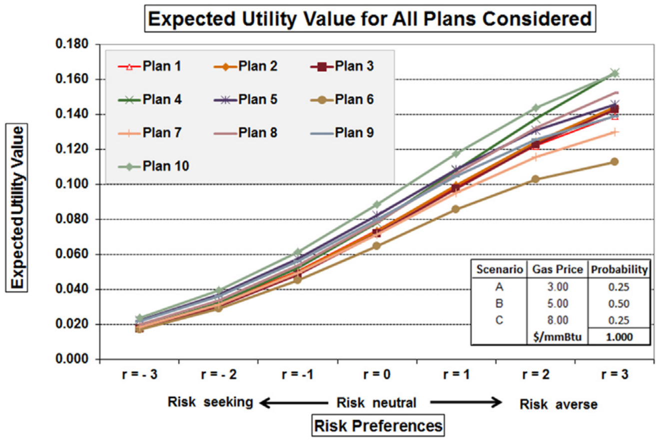

Fig. 5. Expected utility values for all ten potential development plans for a range of risk preferences applying the base case assumptions.

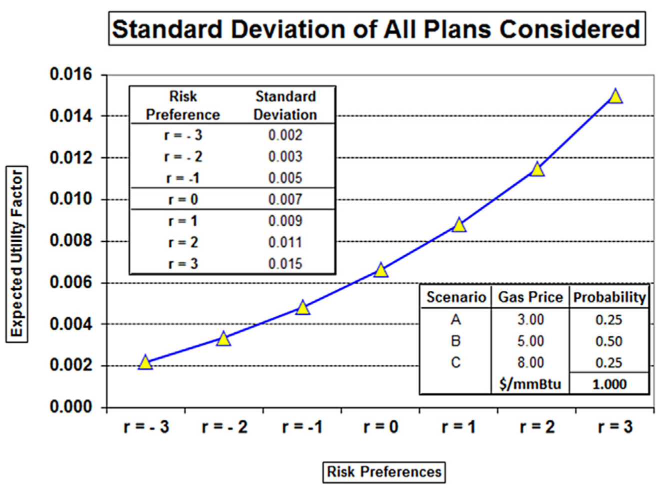

Fig. 6. Standard deviation calculated for the expected utility values of all plans for a given risk preference applying base case assumptions.

field development plan 1, with detailed input economic assumptions, production profiles and capital expenditure profiles for each plan listed in Appendix 1.

The other nine development plans considered involve variations to capital expenditure timing and total investment (e.g. related to timing of drilling of development wells and installation of compression, required in some plans) and consequential variations in the gas production profile. Field operating costs, gas transport costs, fiscal burden and the energy content of the gas are considered constant across all ten potential development plans, but sensitivity analysis could be performed on these variables if necessary. The three key scenarios considered (i.e., A, B and C) involve three different gas sales prices with probabilities of occurrence assigned to each price assumption (Table 7).

The NPVs for each scenario applied to each development plan are evaluated with Equations (10)–(12) to provide utility values and expected utility values (e.g. Table 8) for risk aversion factors varying from −3 (i.e. risk seeking) to +3 (i.e., risk averse).

The expected utility values for all ten plans are shown in Table 9 using the base case scenario assumptions listed in Table 7. Note that for base case assumptions all risk preference case, except for r = 3, Plan 10 is selected as the optimum plan based upon the highest expected utility values. For r = 3 it is Plan 4 that is just selected as the optimum plan. It is interesting to compare the base case NPVs of these two plans:

Plan 10 NPVs: Scenario A = −S42 million; B = 28; C = 129.

Plan 4 NPVs: Scenario A = −S33 million; B = 26; C = 107.

For the base case Plan 10 has the highest scenario C and scenario B NPVs of all ten plans. On the other hand Plan 4 has the least negative value for Scenario A of all ten plans, whereas four plans have less negative values for scenario A than Plan 10. These relationships explain why a highly risk averse decision maker (e.g. r = 3) might select Plan 4 in preference to Plan 10 for the base case assumptions.

The calculated expected utility values for base case assumptions (Table 9) are displayed graphically in Fig. 5.

It is apparent from Table 9 and Fig. 5 that Plans 4 and 10 are more attractive to risk-averse decision makers than risk-prone or risk-neutral decision makers. Also Plan 8 increases its appeal for the more risk-averse investor, moving up to third place in the ranking. Fig. 5 also reveals that there is a much greater spread of expected utility values for the ten plans from the perspective of risk-averse decision makers than from risk-prone decision makers. The standard deviation calculated for the expected utility values of all plans for a given risk preference quantifies that relative dispersion (Fig. 6).

The mean of all risk-seeking expected utility values calculated (i.e., r = −1 to −3) compared to the mean of all risk-averse expected utility values calculated (i.e., r = 1 to 3) and the risk-neutral expected utility values (i.e., EMV calculated with scaled NPVs) are shown in Table 10 and Fig. 7 for all ten plans applying base case assumptions.

It is apparent from Fig. 7 and Table 10 that, for base case assumptions, risk-averse decision makers are more sensitive to the differences between the ten proposed plans, with Plans 4, 5, 8 and 10 appearing more attractive to them. This is similar to the

Table 10
Expected utility values averaged and compared for risk-seeking, risk-averse and risk-neutral decision makers for ten gas field development plans applying base case assumptions listed in Table 7.

Gasfield	Risk seeking (r = −1, r = −2, r = −3)		Risk neutral (r = 0)	Risk averse (r = 1, r = 2, r = 3)	
Plan #	Mean expected utility value	Standard deviation	Expected utility value (scaled EMV)	Mean expected utility value	Standard deviation
1	0.034	0.016	0.074	0.120	0.020
2	0.033	0.016	0.073	0.123	0.022
3	0.032	0.015	0.072	0.121	0.023
4	0.034	0.017	0.078	0.137	0.028
5	0.039	0.018	0.082	0.128	0.019
6	0.030	0.014	0.065	0.100	0.014
7	0.033	0.015	0.072	0.114	0.018
8	0.036	0.017	0.079	0.130	0.023
9	0.038	0.017	0.080	0.123	0.017
10	0.042	0.019	0.089	0.142	0.023

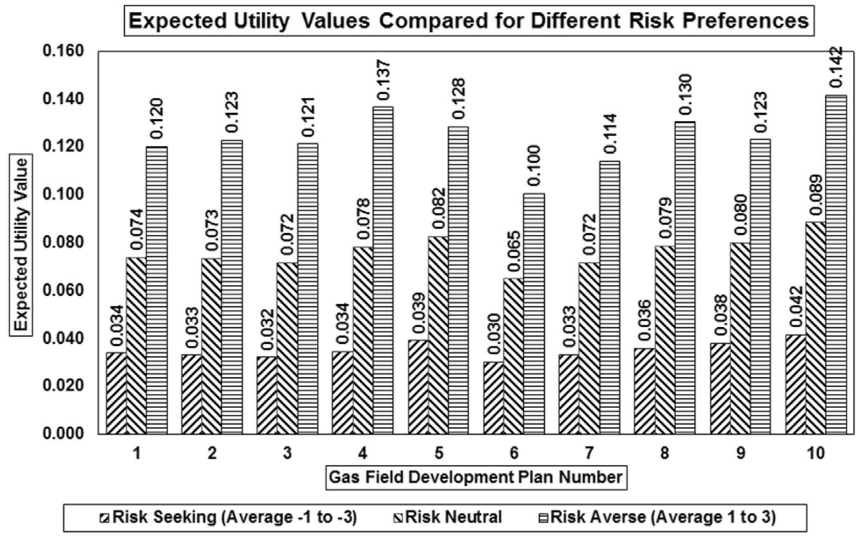

Fig. 7. Expected utility values averaged and compared (Table 10) for risk-seeking, risk-averse and risk-neutral decision makers for ten gas field development plans applying base case assumptions listed in Table 7.

ranking of development plans revealed for risk-neutral decision makers (Fig. 7, Table 10). However, the risk-neutral decision makers value Plan 9 slightly higher than Plan 8. On the other hand, risk-seeking decision makers see little difference between the plans, valuing them all at quite low, as there likely outcomes straddle zero values, but with Plans 5 and 10 appearing more attractive to them.

Table 11 lists the assumptions and expected utility results for nine additional sensitivity cases compared to the base case assumptions (Table 7). These sensitivity cases vary price scenarios, their probabilities, and for cases 4 and 5 the discount rate applied. For cases 5 to 10, all of the risk preferences modelled select Plan 10 as the optimum plan. This is primarily due to the fact that higher gas prices are given more weight leading to high NPVs and favouring cases with high early production, higher cumulative production and delayed capital expenditure (e.g. Plan 10). For the low-gas-price scenario #2 all of the risk preferences modelled select Plan 4, because of its less negative outcomes due to delayed capital investment. Scenarios 3 and 4 show similar optimum case selection to the base case scenario 1, i.e. highly-risk-averse decision makers (r = 3) select Plan 4 whereas other less-risk-averse decision makers select Plan 10.

Table 11
Base case assumptions for the gas field development plan analysis compared with nine sensitivity cases with different gas price, probability and discount rate assumptions.

a) Input variables

Sensitivity analysis	Gas price ($/mmBtu)			Probability of price case			Discount
INPUT variables	A	B	C	A	B	C	Rate
Sensitivity Case #1 (Base)	3.00	5.00	8.00	0.25	0.50	0.25	10.0%
Sensitivity Case #2	2.50	4.00	6.00	0.25	0.50	0.25	10.0%
Sensitivity Case #3	3.00	5.00	15.00	0.25	0.50	0.25	10.0%
Sensitivity Case #4	3.00	5.00	15.00	0.25	0.50	0.25	15.0%
Sensitivity Case #5	3.00	5.00	15.00	0.25	0.50	0.25	5.0%
Sensitivity Case #6	3.00	7.50	18.00	0.35	0.50	0.15	10.0%
Sensitivity Case #7	3.00	7.50	18.00	0.10	0.60	0.30	10.0%
Sensitivity Case #8	3.00	7.50	18.00	0.20	0.40	0.40	10.0%
Sensitivity Case #9	5.00	7.50	10.00	0.25	0.50	0.25	10.0%
Sensitivity Case #10	7.50	10.00	12.00	0.25	0.50	0.25	10.0%

b) Output calculations

Sensitivity analysis	Maximum expected utility value			Standard deviation across plans			Optimum Plan # Selected		
OUTPUT calculations	r = Minus3	r = 0	r = Plus3	r = Minus3	r = 0	r = Plus3	r = Minus3	r = 0	r = Plus3
Sensitivity Case #1 (Base)	0.024	0.089	0.164	0.002	0.007	0.015	10	10	4
Sensitivity Case #2	0.001	−0.005	−0.063	0.001	0.007	0.031	4	4	4
Sensitivity Case #3	0.191	0.236	0.253	0.031	0.016	0.014	10	10	4
Sensitivity Case #4	0.085	0.154	0.181	0.013	0.011	0.015	10	10	4
Sensitivity Case #5	0.553	0.357	0.346	0.100	0.021	0.015	10	10	10
Sensitivity Case #6	0.280	0.278	0.315	0.051	0.019	0.013	10	10	10
Sensitivity Case #7	0.541	0.507	0.625	0.100	0.035	0.016	10	10	10
Sensitivity Case #8	0.693	0.557	0.569	0.131	0.038	0.011	10	10	10
Sensitivity Case #9	0.081	0.280	0.551	0.008	0.018	0.022	10	10	10
Sensitivity Case #10	0.186	0.480	0.781	0.022	0.032	0.026	10	10	10

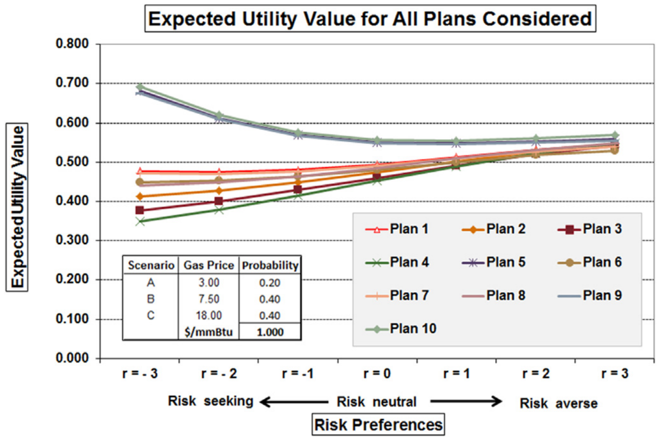

Fig. 8. Expected utility values for all ten potential development plans for a range of risk preferences applying sensitivity case 8 assumptions.

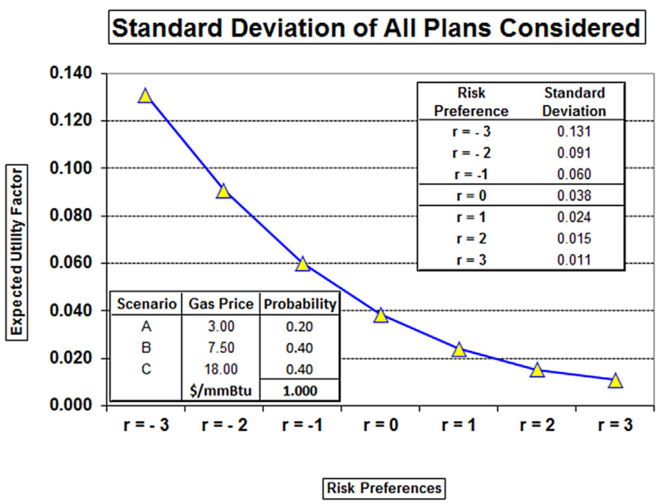

Fig. 9. Standard deviation calculated for the expected utility values of all plans for a given risk preference applying sensitivity case 8 assumptions.

Comparison of the sensitivity cases reveals that the risk-seeking decision makers assign greatest expected utility values to Plan 10 for sensitivity cases 8, 7 and 5 in that descending order. For sensitivity case 6 the highly-risk-averse, risk-neutral and highly-risk-seeking decision makers assign almost the same expected utility value to Plan 10.

Figs. 8–10 provide some further insight to sensitivity case 8 assumptions for the gas field development plan comparison from the perspective of different risk preferences. Note the relative behaviour of risk-averse versus risk-prone decision makers is quite distinct from their responses to base case assumptions.

The development Plans 10 and 5 are clearly favoured by all risk preferences for sensitivity case 8 assumptions, and become much more clearly favoured by risk-seeking decision makers.

In contrast to base case assumptions it is risk-seeking decision makers that see the greatest distinction between the ten development plans, as illustrated by the higher standard deviation of expected utility values for all plans from the perspective of different risk preferences (Fig. 9).

Clearly, analysing field development plan alternatives from the perspective of a range of risk preferences adds some useful insight for decision makers that cannot be gained from discounted cash flow analysis alone.

In addition to gas price the various other factors which determine the gas field development plans such as field operating costs, gas transport costs, fiscal burden and the energy content of the gas also have impacts on the NPV, expected utility value and potential selection of specific development plans. Table 12 lists the assumptions and expected utility results for nine additional sensitivity cases (#11 to #19) compared to the base case assumptions, which vary these other influencing factors.

As should be expected higher field operating costs, gas transportation costs, lower energy content of the gas and higher fiscal burden all tend to lead to Plan 4 being selected over Plan 10 for the risk-neutral and risk-averse decision makers. It is noteworthy in sensitivity case #17 (high-fiscal burden) that the risk-prone decision maker selects Plan8 and the risk-neutral and risk-averse decision makers prefer Plan 4 with Plan 8 in second place (Fig. 11).

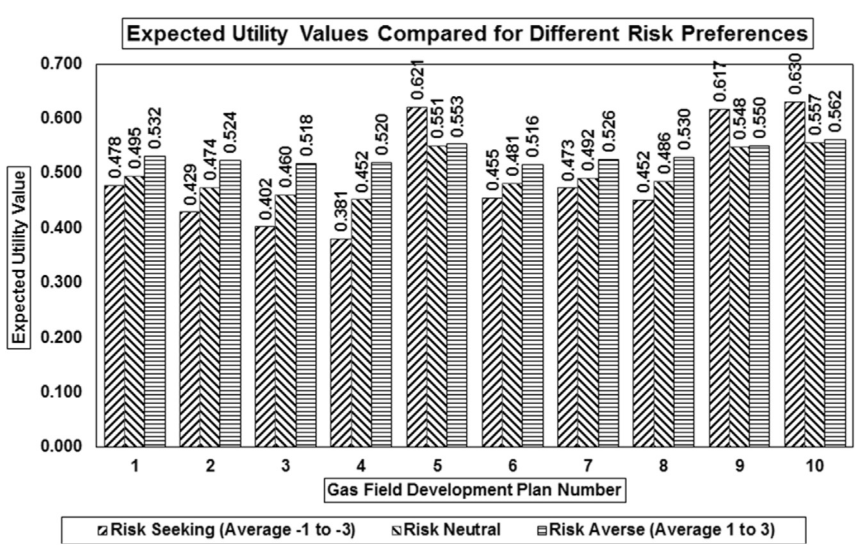

Fig. 10. Expected utility values averaged and compared (Table 10) for risk-seeking, risk-averse and risk-neutral decision makers for ten gas field development plans applying sensitivity **case 8** assumptions.

Table 12
Base case assumptions for the gas field development plan analysis compared with nine sensitivity cases with different field operating costs, gas transportation costs, gas quality, and government fiscal take.

a) Input variables

Sensitivity analysis	Gas price ($/mmBtu)			Probability of price case			Discount rate	Field opex $/mcf	Transport $/mcf	Quality Btu/cf	Gov. take % Profits
INPUT variables	A	B	C	A	B	C					
Sensitivity Case #1 (Base)	3.00	5.00	8.00	0.25	0.50	0.25	10.0%	1.50	1.00	1050	50.0%
Sensitivity Case #2	3.00	5.00	8.00	0.25	0.50	0.25	10.0%	2.00	1.00	1050	50.0%
Sensitivity Case #3	3.00	5.00	8.00	0.25	0.50	0.25	10.0%	0.50	1.00	1050	50.0%
Sensitivity Case #4	3.00	5.00	8.00	0.25	0.50	0.25	10.0%	1.50	2.00	1050	50.0%
Sensitivity Case #5	3.00	5.00	8.00	0.25	0.50	0.25	10.0%	1.50	1.50	1050	50.0%
Sensitivity Case #6	3.00	5.00	8.00	0.25	0.50	0.25	10.0%	1.50	1.00	1000	50.0%
Sensitivity Case #7	3.00	5.00	8.00	0.25	0.50	0.25	10.0%	1.50	1.00	1200	50.0%
Sensitivity Case #8	3.00	5.00	8.00	0.25	0.50	0.25	10.0%	1.00	1.00	1050	75.0%
Sensitivity Case #9	3.00	5.00	8.00	0.25	0.50	0.25	10.0%	1.50	1.00	1050	25.0%
Sensitivity Case #10	3.00	5.00	8.00	0.25	0.50	0.25	10.0%	2.00	2.00	1050	60.0%

b) Output calculations

Sensitivity analysis	Maximum expected utility value			Standard deviation across plans			Optimum Plan # Selected		
OUTPUT calculations	r = Minus3	r = 0	r = Plus3	r = Minus3	r = 0	r = Plus3	r = Minus3	r = 0	r = Plus3
Sensitivity Case #1 (Base)	0.024	0.089	0.164	0.002	0.007	0.015	10	10	4
Sensitivity Case #11	0.014	0.039	0.042	0.001	0.006	0.025	10	4	4
Sensitivity Case #12	0.045	0.170	0.358	0.004	0.011	0.016	10	10	10
Sensitivity Case #13	0.006	−0.001	−0.105	0.001	0.007	0.042	10	4	4
Sensitivity Case #14	0.014	0.039	0.042	0.001	0.006	0.025	10	4	4
Sensitivity Case #15	0.018	0.064	0.117	0.002	0.006	0.018	10	10	4
Sensitivity Case #16	0.043	0.153	0.302	0.004	0.010	0.014	10	10	10
Sensitivity Case #17	0.005	0.023	0.054	0.001	0.005	0.016	8	4	4
Sensitivity Case #18	0.062	0.181	0.303	0.007	0.012	0.016	10	10	10
Sensitivity Case #19	−0.005	−0.058	−0.320	0.001	0.009	0.068	4	4	4

6. Conclusions

Exponential utility functions provide additional insight to risked discounted cash flow analysis for decision makers displaying a range of risk preferences and aiming to select optimum value alternatives from a range of upstream gas and oil assets.

Slightly different approaches are required in calculating meaningful utility values for exploration and development assets. The high-uncertainty, with high potential for failure outcomes of typical exploration assets requires that expected utility values take into account success and failure cases together with their probabilities, but go further than a simple expected monetary value calculation. On the other hand, for field development assets major uncertainties are typically associated with forecasts of market and economic conditions, rather than the presence or absence of the resource. Expected utility factors for a range of risk preferences can be readily calculated, building on net present values of alternative field development plans, to provide useful insight for decision makers in such cases.

Another situation frequently confronting upstream gas and oil decision makers, where utility functions provide useful additional insight to net present values, is where an investor has specific tolerances for loss. The utility value of the same asset to a specific decision maker is likely to vary depending upon whether they are

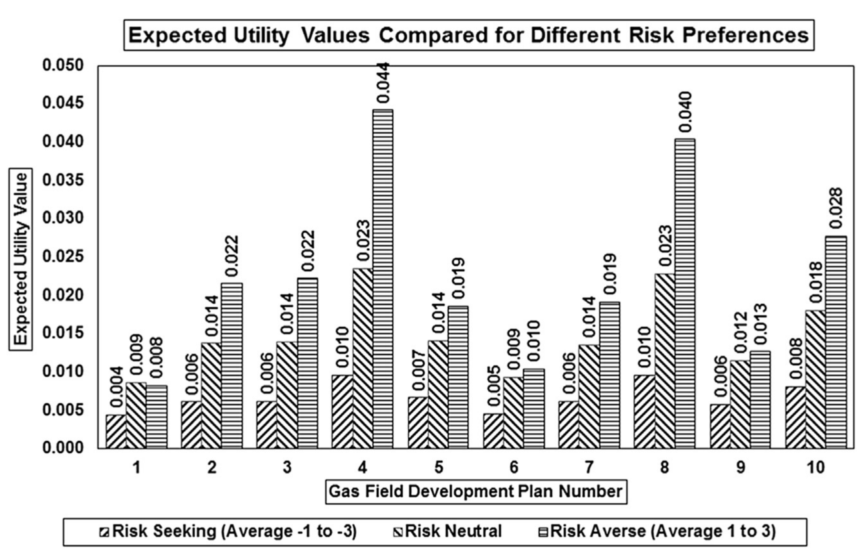

Fig. 11. Expected utility values averaged and compared (Table 12) for risk-seeking, risk-averse and risk-neutral decision makers for ten gas field development plans applying sensitivity **case 17** assumptions.

constrained by relatively-low-risk or relatively-high-risk tolerances. Calculating utility values linked to tolerances for loss can help to establish the relative value of a range of assets under such specific budgetary or strategic constraints.

The simple deterministic examples shown to illustrate the applications of utility functions to exploration, loss tolerance and field development examples can be easily extended to incorporate utility function calculations in more complex stochastic valuation models.

The examples provided in this study make a case for the more-routine consideration of risk preferences and application of exponential utility factors to aid upstream gas and oil decision making.

Appendix

Detailed input assumptions, in addition to those provided in the text, for the gas field development plan assessment presented in Section 5 to enable a reader to reproduce the calculations.

Field operating costs: $1.50/thousand cubic feet (mcf) gas produced.

Gas transport costs: $ 1.0/mcf.

Gas quality: 1050 Btu/cubic foot.

Annual discount rate: 10% (except for sensitivity cases 4 and 5).

Simple fiscal model: 50% government take of profits. Profits calculated as operating cash flow less an annual depreciation deduction of 10% of cumulative capital expenditure for the plan.

Production profiles for each plan (Appendix Table 1):

Year	Gas production (Bcf/Year)									
	Plan 1	Plan 2	Plan 3	Plan 4	Plan 5	Plan 6	Plan 7	Plans 8	Plan 9	Plan 10
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	15.0	10.0	5.0	5.0	15.0	10.0	5.0	5.0	15.0	15.0
4	15.0	10.0	10.0	5.0	15.0	10.0	10.0	5.0	15.0	15.0
5	15.0	15.0	15.0	10.0	15.0	15.0	15.0	10.0	15.0	15.0
6	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
7	15.0	15.0	15.0	20.0	15.0	15.0	15.0	20.0	15.0	15.0
8	10.0	15.0	15.0	15.0	15.0	15.0	20.0	20.0	15.0	15.0
9	10.0	10.0	15.0	15.0	15.0	15.0	15.0	20.0	15.0	15.0
10	5.0	10.0	10.0	15.0	10.0	10.0	15.0	15.0	10.0	10.0
Total	**100.0**	**100.0**	**100.0**	**100.0**	**115.0**	**105.0**	**110.0**	**110.0**	**115.0**	**115.0**

Capital expenditure profiles for each plan (Appendix Table 2):

Year	Capital expenditure (Capex) ($/millions)									
	Plan 1	Plan 2	Plan 3	Plan 4	Plan 5	Plan 6	Plan 7	Plan 8	Plan 9	Plan 10
1	30.0	30.0	30.0	20.0	30.0	30.0	30.0	20.0	30.0	30.0
2	60.0	20.0	15.0	10.0	60.0	20.0	15.0	10.0	60.0	60.0
3	0.0	20.0	15.0	10.0	0.0	20.0	15.0	10.0	0.0	0.0
4	0.0	20.0	15.0	10.0	0.0	20.0	15.0	10.0	0.0	0.0
5	0.0	0.0	15.0	20.0	0.0	0.0	15.0	20.0	0.0	0.0
6	0.0	0.0	0.0	20.0	0.0	0.0	0.0	20.0	20.0	0.0
7	0.0	0.0	0.0	0.0	20.0	20.0	20.0	20.0	0.0	5.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	**90.0**	**90.0**	**90.0**	**90.0**	**110.0**	**110.0**	**110.0**	**110.0**	**110.0**	**105.0**

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